

# The stochastic parameterization of non-orographic GWs tested in the LMDz GCM

F. Lott, P. Maury and L. Guez

- 1) Motivation and Formalism
- 2) Implementation in the LMDz GCM
- 3) Impact on the QBO and on the planetary-scale equatorial waves
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# 1) Motivation and formalism

*Classical arguments: see Palmer et al. 2005, Shutts and Palmer 2007, for the GWs: Piani et al. (2005, globally spectral scheme) and Eckeman (2011, multiwaves scheme)*

1) The spatial steps  $\Delta x$  and  $\Delta y$  of the unresolved waves is not a well defined concept (even though they are probably related to the model gridscales  $\delta x \delta y$ ). The time scale of the GWs life cycle  $\Delta t$  is certainly larger than the time step ( $\delta t$ ) of the model, and is also not well defined.

2) The mesoscale dynamics producing GWs is not well predictable (for the mountain gravity waves see Doyle et al. MWR 11).

These calls for an extension of the concept of triple Fourier series, which is at the basis of the subgrid scale waves parameterization to that of stochastic series:

$$w' = \sum_{n=1}^{\infty} C_n w'_n \quad \text{where} \quad \sum_{n=1}^{\infty} C_n^2 = 1$$

The  $C'_n$ s generalised the intermittency coefficients of Alexander and Dunkerton (1995), and used in Beres et al. (2005).

# 1) Motivation and formalism

For the  $w'_n$  we next use the linear WKB theory of hydrostatic monochromatic waves, and treat their breaking as if each  $w'_n$  was doing the entire wave field (using Lindzen (1982)'s criteria for instance): they are viewed as independent realizations.

$$w'_n = \Re \left\{ \hat{w}_n(z) e^{z/2H} e^{i(k_n x + l_n y - \omega_n t)} \right\}$$

$\hat{w}_n, k_n, l_n, \omega_n$  will all be chosen randomly

Passage from one level to the next:

$$\hat{w}_n(z + \delta z) = \hat{w}_n(z) \sqrt{\frac{m(z)}{m(z + \delta z)}} e^{-i \int_z^{z+\delta z} m(z') dz'} \quad \text{where} \quad m = \frac{N |\vec{k}_n|}{\Omega} \quad \text{and} \quad \Omega = \omega_n - \vec{k}_n \cdot \vec{u}$$

$$\text{Saturation: } |\hat{w}_n| \leq w_s = \frac{\Omega^2}{|\vec{k}_n| N} e^{-z/2H} S_c \frac{|\vec{k}_n|}{k^*}, \quad S_c, k^* : \text{Tunable parameters}$$

$$\text{EP-flux: } \vec{F}_n^z = \Re \left\{ \rho_r \frac{\vec{u}_n \hat{w}_n^*}{2} \right\} = \rho_r \frac{\vec{k}_n}{2 |\vec{k}_n|^2} m(z) |\hat{w}_n(z)|^2$$

All in this page may be over-simple but can easily be modified to include rotation, Non-hydrostatic effects, more sophisticated breaking criteria (Richardson dependent, ...), or more sophisticated wave packets

## 2) Implementation in the LMDz-GCM

Few waves (say  $M=8$ ) are launched at each physical time step ( $\delta t=30\text{mn}$ ), but their effect is redistributed over a longer time scale ( $\Delta t=1\text{day}$ ), so around 400 waves are active at the same time:

This excellent spectral resolution is the major benefit of the method.

$M$  and  $\Delta t$  are two extra tunable parameters (Could be random numbers as well)

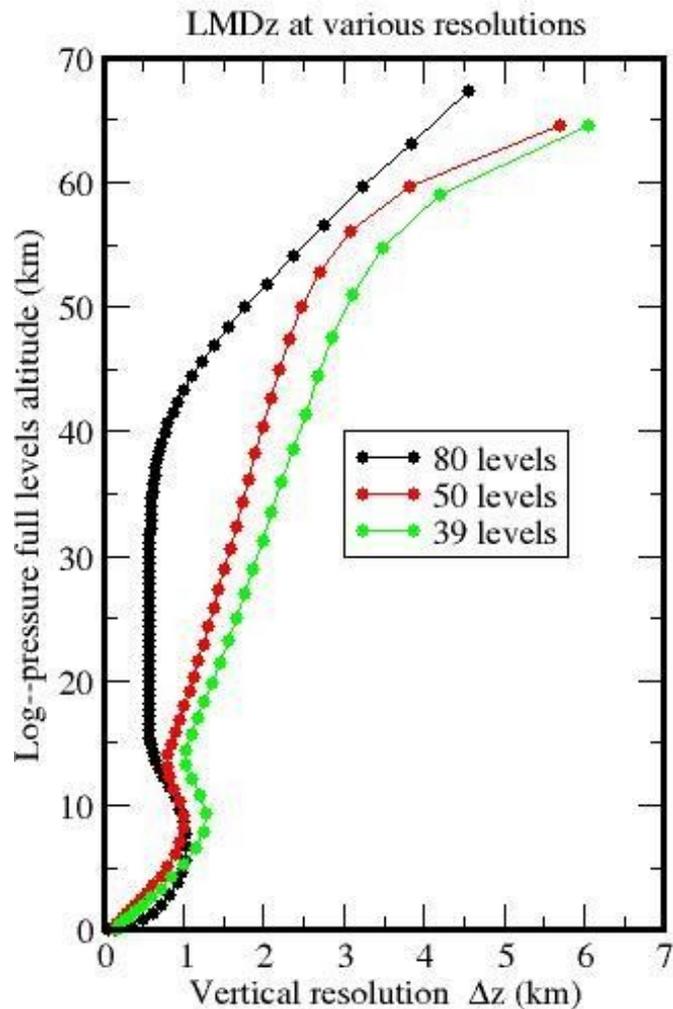
The redistribution is done via an AR-1 protocol which forces to keep the GWs tendency, e.g. 2 other 3D fields that will be needed at the re-start of the model:

$$\left(\frac{\partial \vec{u}}{\partial t}\right)_{GWs}^{t+\delta t} = \frac{\Delta t - \delta t}{\Delta t} \left(\frac{\partial \vec{u}}{\partial t}\right)_{GWs}^t + \frac{\delta t}{M \Delta t} \frac{\sum_{n'=1}^M 1}{\rho} \frac{\partial \vec{F}_n^z}{\partial z}$$

At each time we promote  $M$  new waves and degrade the probability of all the others by the AR-1 factor  $(\Delta t - \delta t)/\delta t$  (they loose their AAA!):

$$C_n^2 = \left(\frac{\Delta t - \delta t}{\Delta t}\right) \frac{\delta t}{M \Delta t}, \quad \vec{F} = \sum_{n=1}^{\infty} C_n^2 \vec{F}_n$$

## 2) Implementation in the LMDz-GCM



### Model set-up:

The model vertical resolution is increased up to 80 levels compared to the 50 stratospheric level version documented in Lott et al. 2005, or to the 39 levels used for CMIP 5 (Maury et al. 2011)

Horizontal resolution, 96x95 grid.

### GWs setup:

Launch altitude of the waves 800hPa,

Random choices (with equal probabilities between The specified bounds):

$F^z$  at launched altitude between 0 and 30mPa

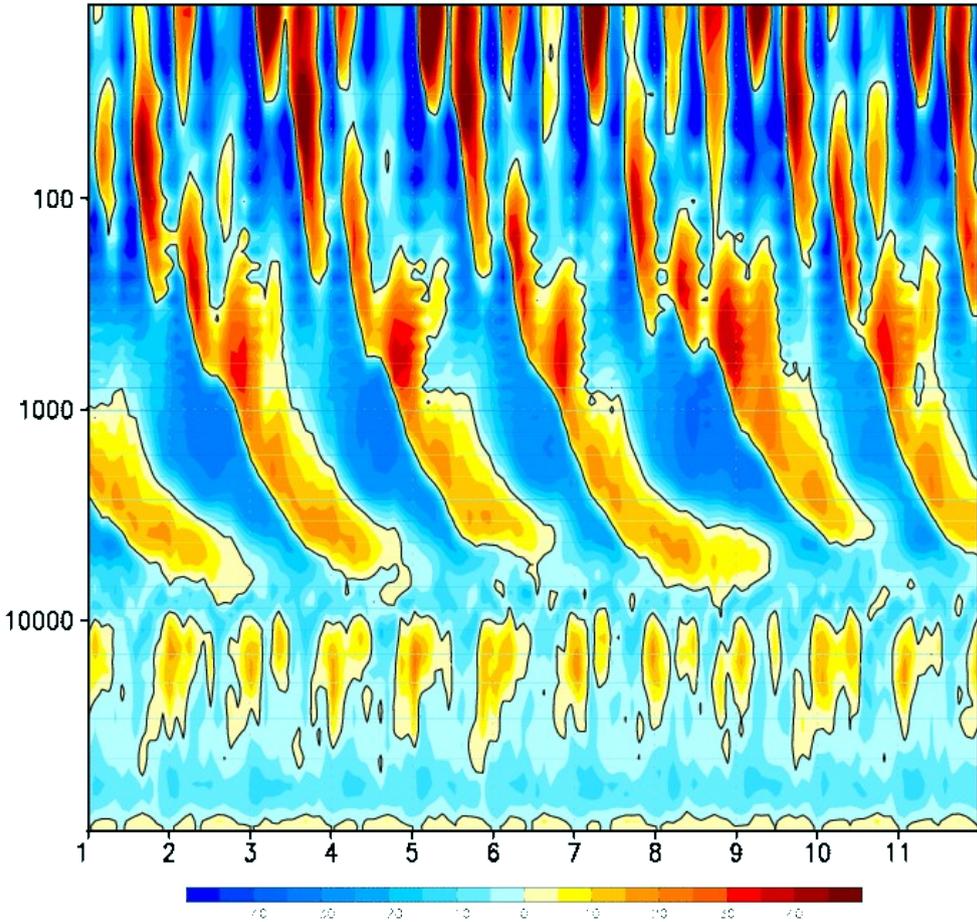
$k$  between  $k_{\min}=1/(1\text{km})$  and  $k^*=1/(100\text{km})$ .

Absolute phase speed between -30m/s and 30m/s

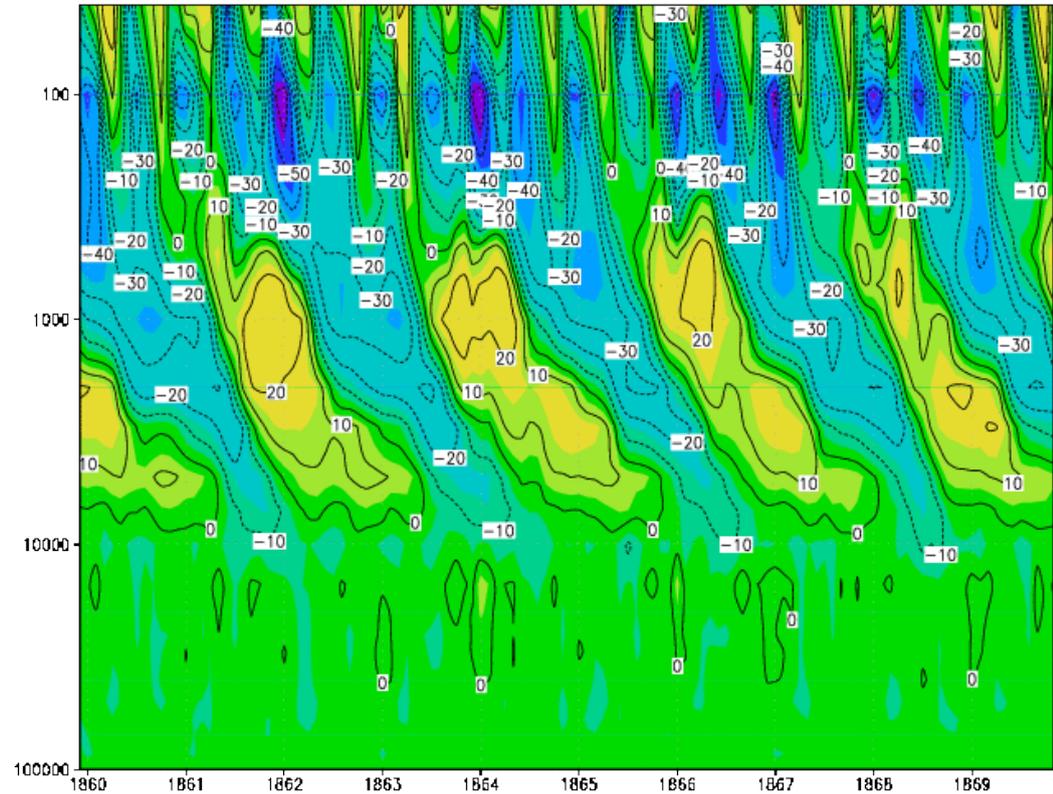
No extensive tunings have been done on line so far

### 3) Impact on the QBO and the equatorial waves

equatorial zonal mean zonal wind



U equ, Historical HadGEM2-CC



All that to produce a BO!

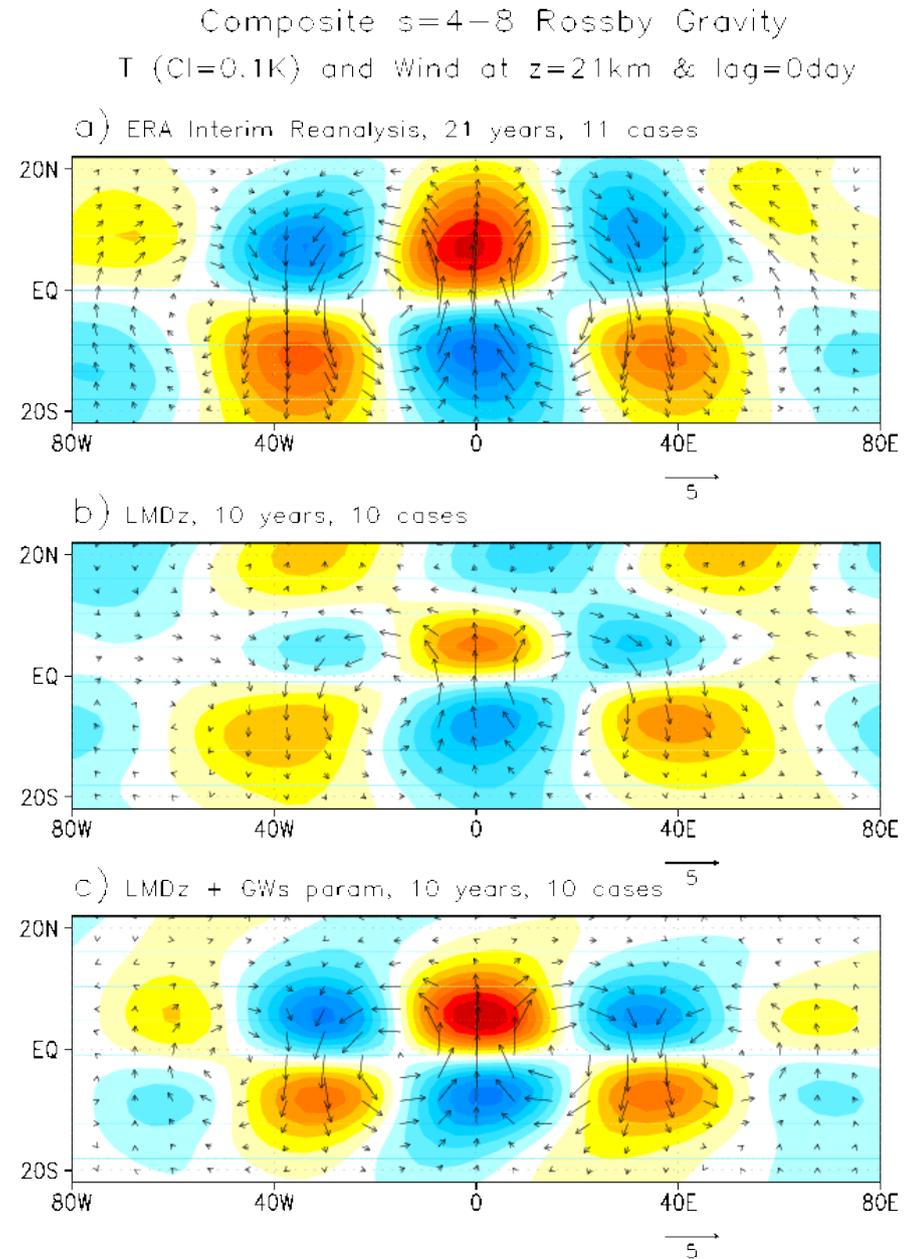
Lott, Guez, Maury (2012)

### 3) Impact on the QBO and the equatorial waves

When you start to have positive zonal winds, the planetary scale Yanai wave is much improved

(the composite method is described in Lott et al. 2009, applied to model datas in Maury et al. 2011)

Extraction of the stratospheric Eq. Waves in CMIP 5 models, work done in the COMBINE EU-FP7 project, and in collaboration with MPI-M, and UK-Met Office



### 3) Impact on the QBO and the equatorial waves

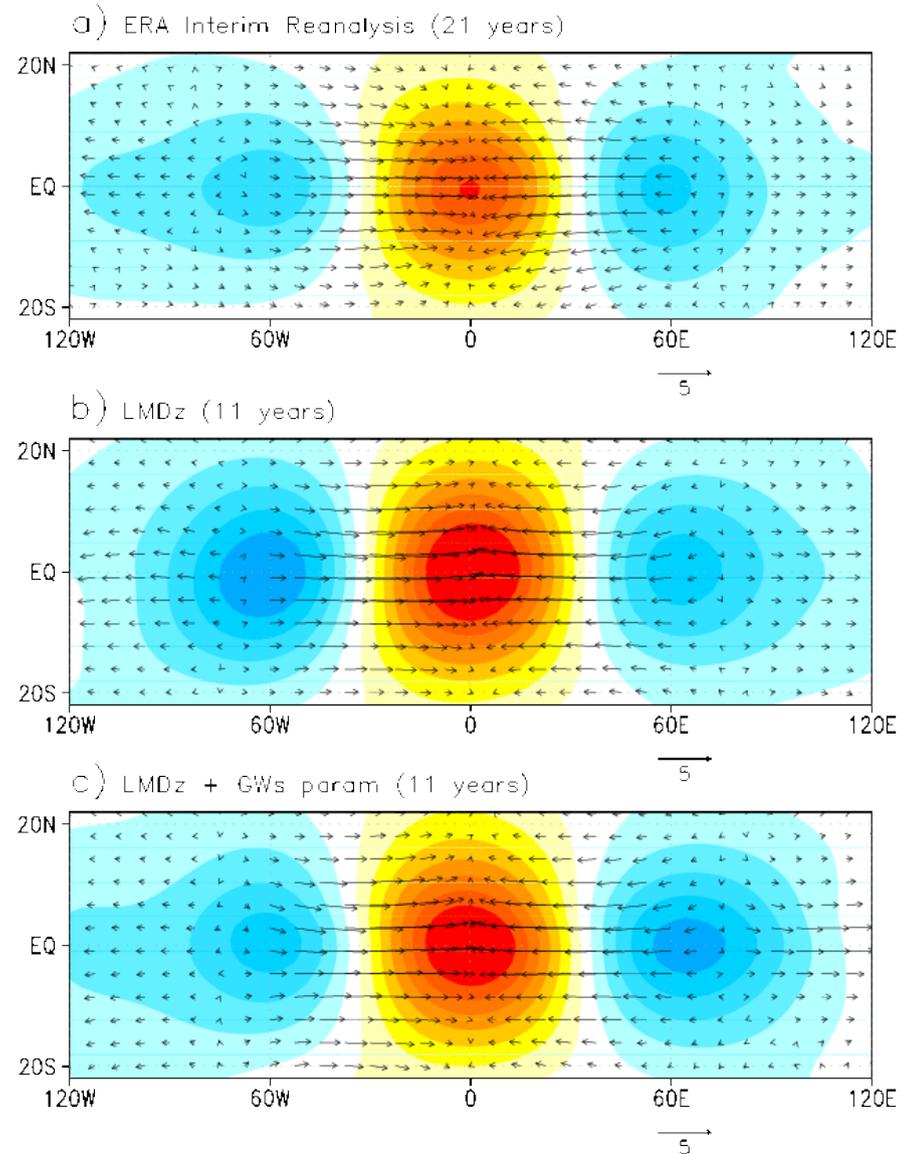
The Kelvin waves is almost unchanged, because without gravity Waves the model zonal mean zonal wind are always negative  
In the low stratosphere (the KW are very far from critical level situations)

Theoretical problem:  
Why in all models the planetary scale Kws always have the same amplitude  
( $u' < c$ , so shallow water prediction like In Boyd~1980, or LeSommer et al. 2005 do not apply)

It is hard to find a theoretical problem that is relevant to the real world, so here you have one.

Composite Kelvin waves

T (CI=0.4K) and Wind at z=21km

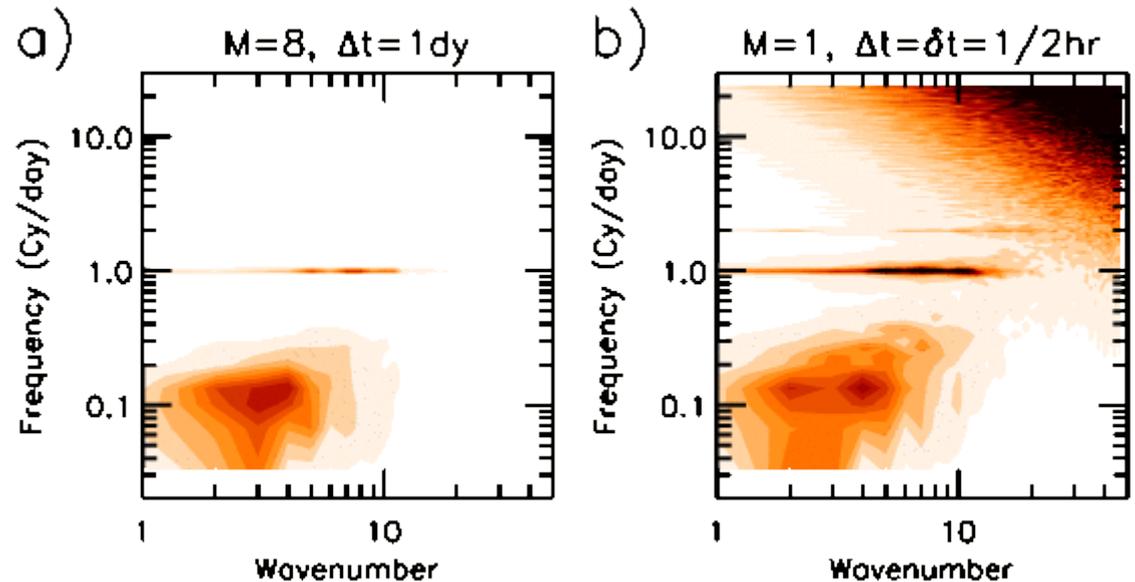


### 3) Impact on the QBO and the equatorial waves

Variability injection:

A lot when  $M=1$ ,  $\delta t=\Delta t=6\text{hr}$   
(Eckerman 2011),  
and near the model truncature,  
which is not a good think.

This is much reduced in our  
Scheme with  $M=8$ ,  $\Delta t=1\text{dy}$ ,  
but this can be controlled  
and extended to other  
parameterizations to inject  
variabilty at the scales which  
are not well resolved



Stochastic methods have fundamental  
Justifications (as given here) but can also  
be used to inject Kinetic energy  
in the tails of the model spectra

You made need to share information  
between adjacent grid cells which is a difficulty  
in our massively parallel environments

(see Kinetic energy Backscatter, Shutts 2005)

#### **4) Advantages, limits and relation with the sources**

Advantages: Very high spectral resolution, which is good for the treatments of critical levels (an important aspect of the QBO dynamics).

Very cheap cost (but about the same cost as the Hines (1997) Parameterization schemes).

Easy to relate to the convective sources  
(see the linear formalism in Beres et al. 2005)

More fundamental: there is no reason to treat the mesoscale dynamics as predictable from the large-scale flow and using few tunable parameters

Defect: What is true for critical levels is not for the waves breaking far from them, linear theory is not adapted to describe it. In this sense, the globally spectral methods (Hines (1997), Warner and McIntyre (1997)) are may be more adapted.

But: Imposing spectral shapes at all time is also quite incorrect since:  
Spectra are the superposition of individual periodograms, each realisation is not likely to have a Fourier representation that resemble to the Spectra

#### **Can we reconcile the spectral methods and the multiwave methods?**

This is an other theoretical problem of big interest, again there is a lot of literature but little is of practical interest

Observations with constant level balloons often show that the waves in the stratosphere have quite narrowbanded spectra (Hertzog et al. 2008), and are highly intermittent.

## 4) Advantages, limits and relation with the sources

Relation with the convective sources (as in Beres et al. (2005)'s linear approach):

$$T \frac{ds}{dt} = c_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dp}{dt} = \frac{dW}{dz} = Q$$

Q is the heating in Watts/m<sup>3</sup>  
W is the heat flux (this is just to emphasize that the vertical distribution is an issue).

$$\left( \partial_t + \vec{u} \cdot \vec{\nabla} \right) \Phi'_z + N^2 w' = J' = \frac{R}{\rho C_p H} Q' = J'$$

Develop the heating in our pseudo-Fourier (stochastic series)

$$J' = \sum_{n=1}^{\infty} C_n J'_n = \sum_{n=1}^{\infty} C_n \hat{J}_n(z) e^{z/2H} e^{i(k_{nx} + l_{ny} - \omega_n t)}$$

Let's impose a vertical distribution and consider that the variance of the subgrid scale distribution of the heating is proportional to the square gridscale heating (I did not think much about the statistic yet, but this seems more or less

OK to start with since precipis are a 0/1 process)

$$J = J_c \frac{e^{-z/\Delta Z^2}}{\pi \Delta z}$$

$$J_c \approx \text{Random}(0,1) \frac{R L_v}{C_p H} Pr$$

Pr: Precipitation or other measure of the grid scale diabatic heating

## 4) Advantages, limits and relation with the sources

$$\hat{w}_{zz} + \frac{k^2 N^2}{\Omega^2} \hat{w} = \frac{k^2}{\Omega^2} \hat{J}$$

Hydrostatic, WKB, Boussinesq approximations!

Resolution with a Green function approach:

$$\hat{w} = \int_{-\infty}^{+\infty} \hat{w}_0(z') W(z-z') dz'$$

$$\hat{w}_0(z') = J_c \frac{k^2 e^{-z'^2/\Delta Z^2}}{\sqrt{\pi} \Delta Z \Omega^2}$$

and

$$W_{zz} + \frac{k^2 N^2}{\Omega^2} W = \delta(z-z')$$

If we can consider that  $z > z'$ :

$$\hat{w} \approx \int_{-\infty}^{+\infty} i \frac{J_c |\vec{k}|}{\sqrt{\pi} \Delta z \Omega} e^{-z'^2/\Delta z^2} e^{-\Im(z-z')} dz' = i \frac{J_c |\vec{k}|}{4 N \Omega} e^{-m^2 \Delta z^2/4} e^{-imz}$$

Work done in EMBRACE (EU-FP7 project), and in collaboration with the UK Met-Office