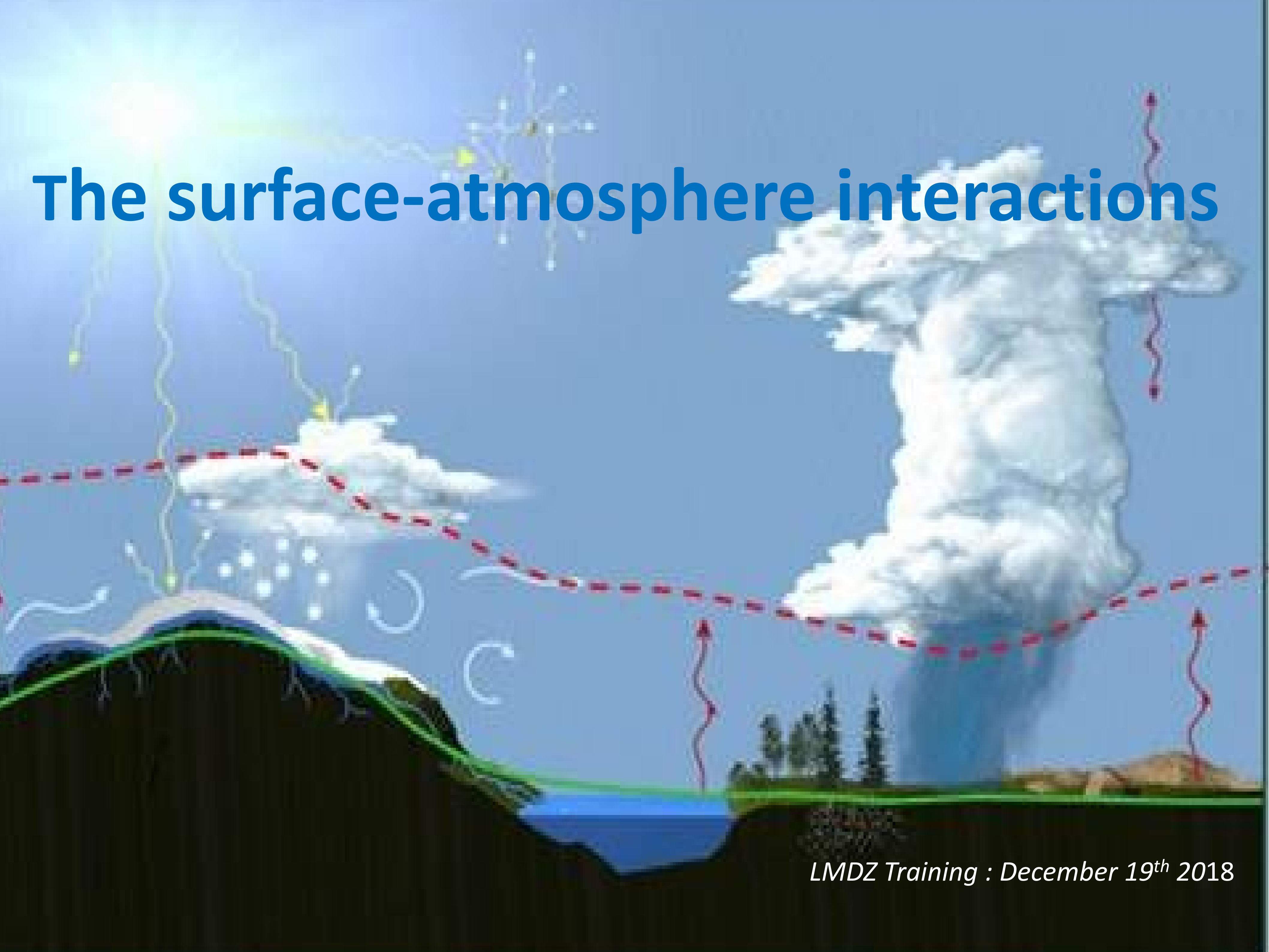
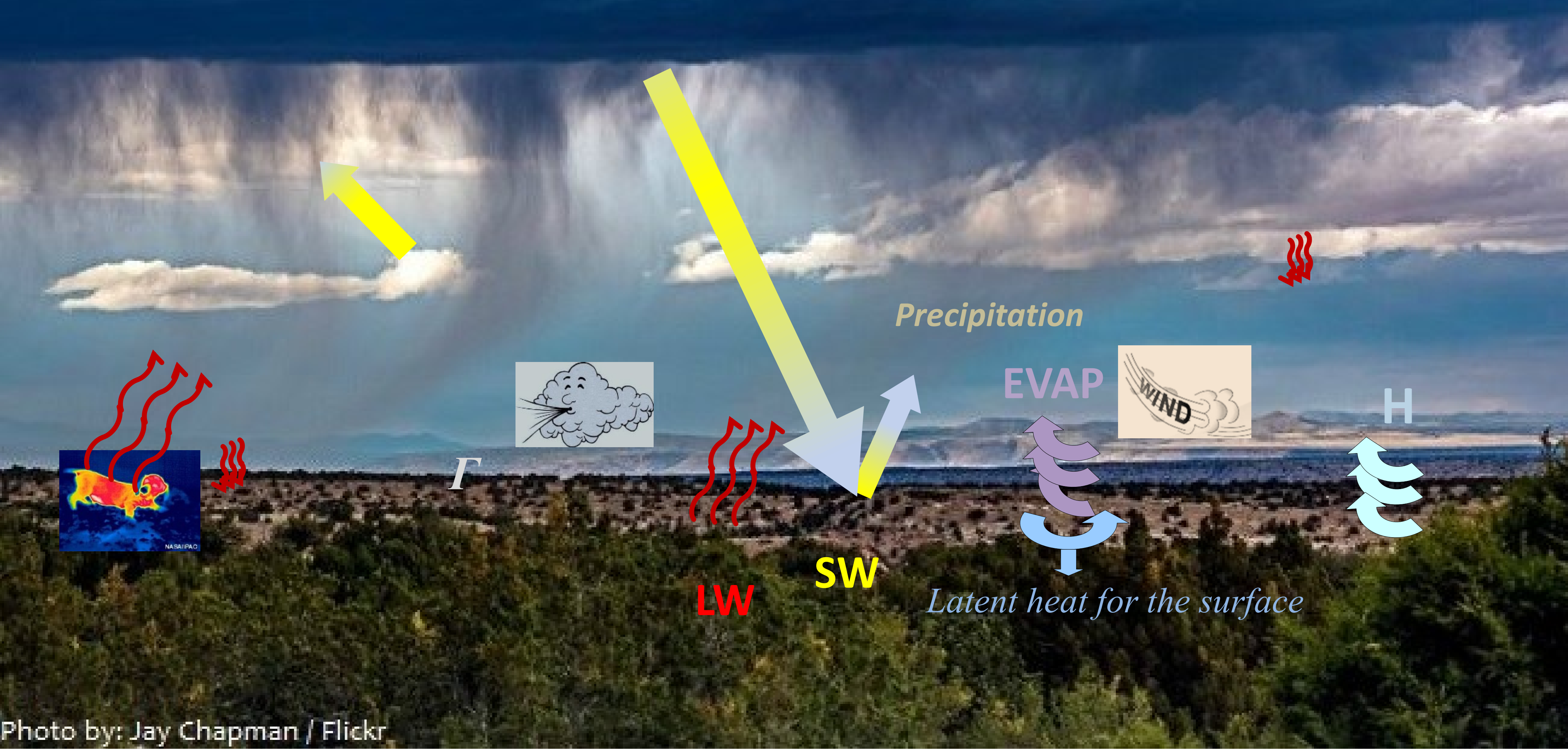
The surface-atmosphere interactions



LMDZ Training : December 19th 2018

Processes involved



The **atmosphere and the surface are coupled** through *turbulence* (in boundary layer) and *radiation* (SW and LW). Currently, there is no direct influence of the surface to other parametrizations.

The surface “receive” precipitation from the atmosphere (no direct feedback).

Atmosphere-surface interactions

The **atmosphere and the surface are coupled** through *turbulence* (in the boundary layer) and *radiation* (SW and LW). Currently, there are no direct influence of the surface to other parametrizations.

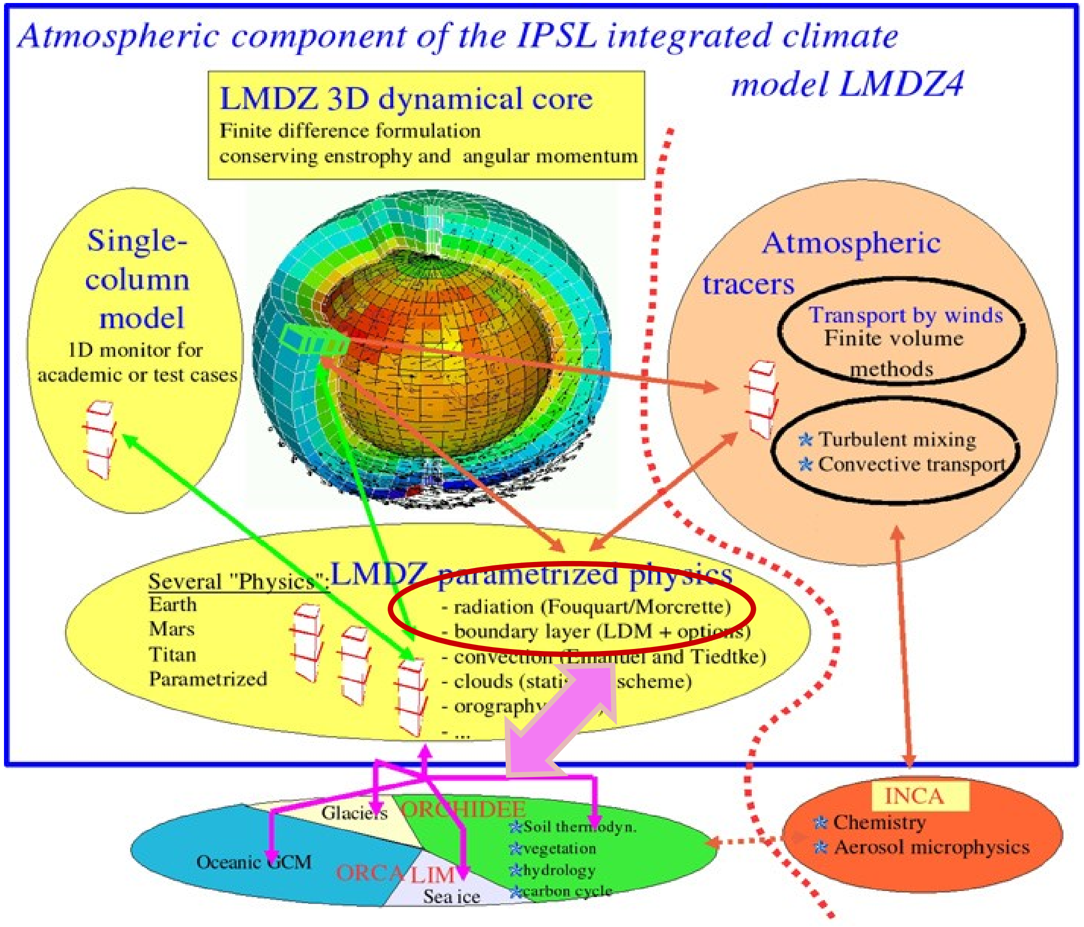
The surface “receive” precipitation from the atmosphere (no direct feedback).

The surface impacts the atmosphere via the orography (factors constant with time), roughness, albedo emissivity

In LMDZ:
Each surface grid can be decomposed in a Maximum of 4 sub-grid of different type:
land (_ter), continental ice (_lic),
open ocean (_oce) and sea_ice (_sic)

Radiation at the surface depends on mean surface properties (albedo, emissivity)

Turbulent diffusion depends on local sub-grid droperties but each sub-surface sees the same atmosphere



Turbulent diffusion (pbl_surface)

- Change of a variable X with the time due to the turbulent transport (continuity) :

$$\frac{\partial X}{\partial t} = -\frac{1}{\rho}\frac{\partial \Phi}{\partial z} = -\frac{\partial \Phi}{\mathrm{m_l}}$$

m_l = mass per surface unit (kg/m^2)

X= specific humidity, momentum, moist static energy, tracers

$$\Phi = -\rho k_z \frac{\partial X}{\partial z}$$

k_z Diffusion coefficient (m^{2}s^{-1})

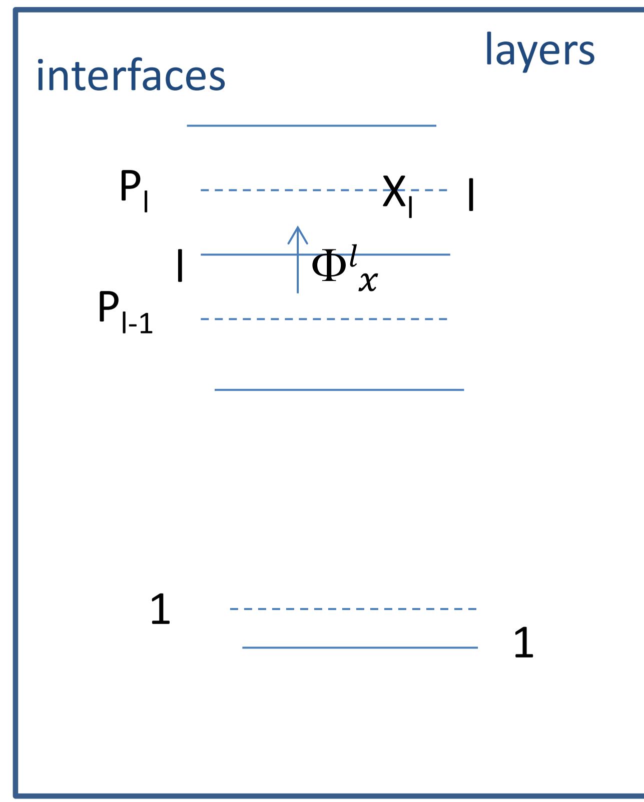
Φ: upward positive

- Vertical discretization

$$\boxed{\Phi^l = -K_l\,(X_l - X_{l-1})}$$

$\delta P = (P_{l-1} - P_l) = \rho g \delta z = m_l g$

$$K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l}$$



vertical discretization

$$\Phi^{l}{}_{x} = -K_l\,(X_l - X_{l-1})$$

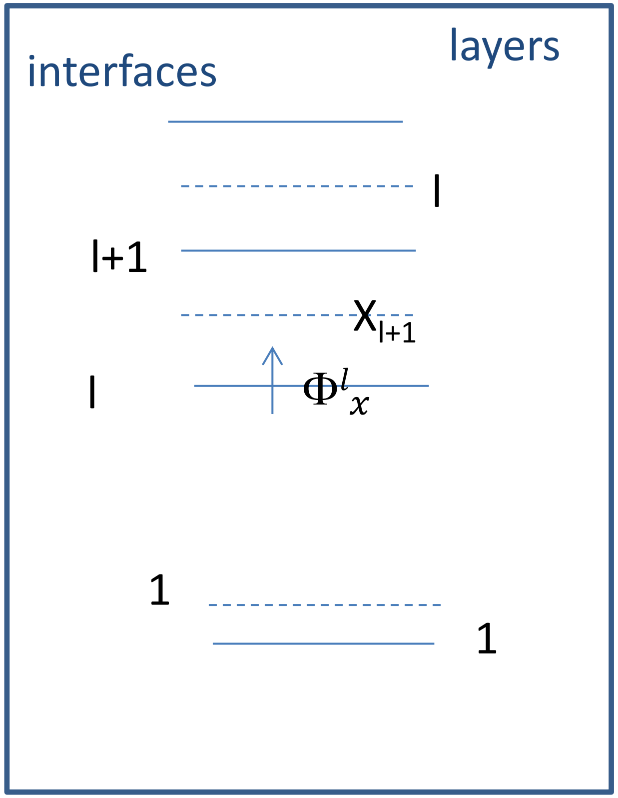
$$m_l \frac{X_l(t+\delta t) - X_l(t)}{\delta t} = \phi_l(t+\delta t) - \phi_{l+1}(t+\delta t)$$

$$m_l \frac{X_l - X_l^0}{\delta t} = \phi_l - \phi_{l+1}$$ with $X_l = X_l(t+\delta t)$, $X_l^0 = X_l(t)$

$$m_l \frac{X_l - X_l^0}{\delta t} = K_{l+1}(X_{l+1} - X_l) - K_l(X_l - X_{l-1})$$

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

$$\boxed{-K_l X_{l-1} + \left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l + K_{l+1} X_{l+1} = \frac{m_l}{\delta t} X_l^0}$$



Tri-diagonal system that can be solved for the vector X

Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$\boxed{\left(\delta P_l + R_{l+1}^X + R_l^X\right) X_l = \delta P_l \; X_l^0 + R_{l+1}^X \; X_{l+1} + R_l^X \; X_{l-1}} \quad (2 \leq l < n)$$

$$\text{with } R_l^X = g \delta t K_l$$

At the top (l=n, $\Phi_n = 0$)

$$\boxed{\left(\delta P_n + R_n^X\right) X_n = \delta P_n \; X_n^0 + R_n^X \; X_{n-1}}$$

At the bottom: (l=1) : $m_1 \frac{X_1 - X^0_1}{\delta t} = \Phi^1{}_x - \Phi^2{}_x$

$$m_1 \frac{X_1 - X_1^0}{\delta t} = K_2(X_2 - X_1) - F_1^X$$

$$\boxed{\left(\delta P_1 + R_1^X\right) X_1 = \delta P_1 \; X_1^0 + R_2^X \; X_2 - g \delta t F_1^X}$$

With F_1^X : flux of X at the bottom of the first layer (i.e. between the surface and the atmosphere), positive downward.

$$K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l}$$

Solving the tridiagonal system

Starting from top:

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n \; X_n^0 + R_n^X \; X_{n-1}$$

can be written as:

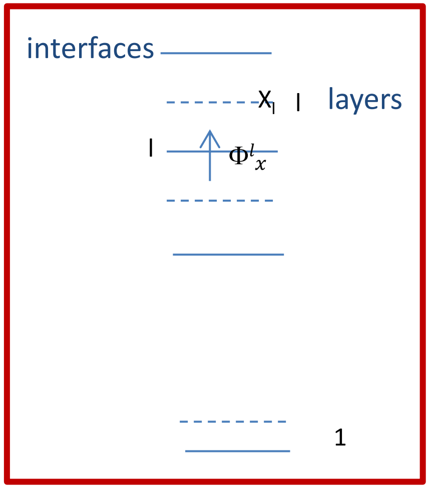
$$\boxed{X_n = C_n^X + D_n^X X_{n-1}}$$

with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$

with $R_l^X = g\delta t K_l$

$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$



Solving the tridiagonal system

$$\boxed{X_{l+1} = C^X_{l+1} + D^X_{l+1} X_l}$$

$$\left(\delta P_l + R^X_{l+1} + R^X_l\right) X_l \quad = \quad \delta P_l \; X^0_l + R^X_{l+1} X_{l+1} + R^X_l \;\; X_{l-1} \quad (2 \le l < n$$

$$\left(\delta P_l + R^X_{l+1}\left(1 - D^X_{l+1}\right) + R^X_l\right) X_l \quad = \quad \delta P_l \; X^0_l + R^X_{l+1} \; C^X_{l+1} + R^X_l \;\; X_{l-1}$$

with $R^X_l = g\delta t K_l$

So we obtain by reccurence:

$$\boxed{X_l = C^X_l + D^X_l X_{l-1}} \qquad (2 \le l \le n)$$

with, for $(2 \le l < n)$

$$\left\{\begin{aligned} C^X_l &= \frac{X^0_l \delta P_l + R^X_{l+1} C^X_{l+1}}{\delta P_l + R^X_l + R^X_{l+1}(1 - D^X_{l+1})} \\ D^X_l &= \frac{R^X_l}{\delta P_l + R^X_l + R^X_{l+1}(1 - D^X_{l+1})} \end{aligned}\right.$$

depend only on properties in the layers above and the variables at the previous time step.

Solving the tridiagonal system

At the bottom of the boundary layer $X_2 = C_2^X + D_2^X X_1$

$$\left(\delta P_1 + R_2^X\right) X_1 = \delta P_1 \; X_1^0 + R_2^X \; X_2 - g\delta t F_1^X$$

replacing X_2 in the equation above:

$$\boxed{X_1 = A_1^X + B_1^X.F_1^X.\delta t}$$

with

$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X(1 - D_2^X)}$$

$$B_1^X = \frac{-g}{\delta P_1 + R_2^X(1 - D_2^X)}$$

Solving the tridiagonal system

interfaces layers

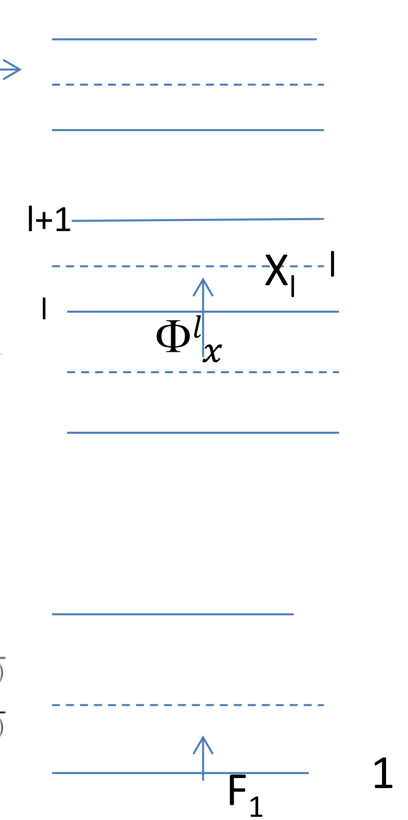
$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$

$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$

downhill

$$C_l^X = \frac{X_l^0 \delta P_l + R_{l+1}^X C_{l+1}^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

$$D_l^X = \frac{R_l^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$



$$X_l = C_l^X + D_l^X X_{l-1}$$

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$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$

?

$$X_1 = A_1^X + B_1^X F_1^X \delta t$$

X= wind, enthalpie, specific humidity, tracers

F_1^x (flux of water mass, flux of heat, flux of momentum) is either prescribed or computed for each sub-surface

Once F_1^x is known, the X_i can be computed from the first layer to the top of the PBL

Coupling with the surface : Compute F_x^1

Depends on the vertical diffusion scheme

$$F^1_X = \text{Bulk formula} = \rho\, C_d^x |V| (X_1 - X_s) = \frac{X_1 - A_1}{B_1 \delta t}$$

C_d^x drag coefficient (Monin Obukhov, constant flux in the surface layer) depends on

- roughness lenghts (gustiness, vegetation),
- Richardson number (boundary layer stability)
- Formulation depends on the sub-surface type

routine cdrag.F90

Once X_s is known , X_1 and F^1_X are known

Coupling with the surface : compute T_s and F_1^T (sensible heat flux)

Case of the continental surface and the temperature

- Heat conduction in the soil: Diffusion equation :

$$\begin{cases} \Phi_T = -\lambda \dfrac{\partial T}{\partial z} \\ \dfrac{\partial T}{\partial t} = -\dfrac{1}{C}\dfrac{\partial \Phi_T}{\partial z} \end{cases}$$

λ= thermal diffusivity

C = thermal capacity

Boundary conditions:

✓ bottom : $\Phi = 0$

✓ top: Continuity of the fluxes and the temperature between sub-surface and atmosphere

$$SW_{net} + LW_d - \varepsilon\underbrace{\sigma T_s^4 + H + L + \Phi_0}_{\text{depend on Ts}} = 0$$

$$L = \beta\rho C_p V C_d\,(q_s(T_s) - q_1) = \rho C_p V C_d\,(q_{surf} - q_1)$$

$$H = -\rho C_p V C_d\,(T_1 - T_s)$$

Coupling with the surface : compute T_s and F_1^T (sensible heat flux)

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Boundary conditions:

- ✓ bottom : $\Phi = 0$
- ✓ top: Continuity of the fluxes and the temperature between sub-surface and atmosphere

$$SW_{net} + LW_d \underbrace{- \varepsilon\sigma T_s^4 + H + L + \Phi_0}_{\text{depend on Ts}} = 0$$

$$H = \beta V C_d (q_1 - q_s(T_s))$$

$$L = \rho V C_d (T_1 - T_s)$$

Vertical discretization and time discretization of C

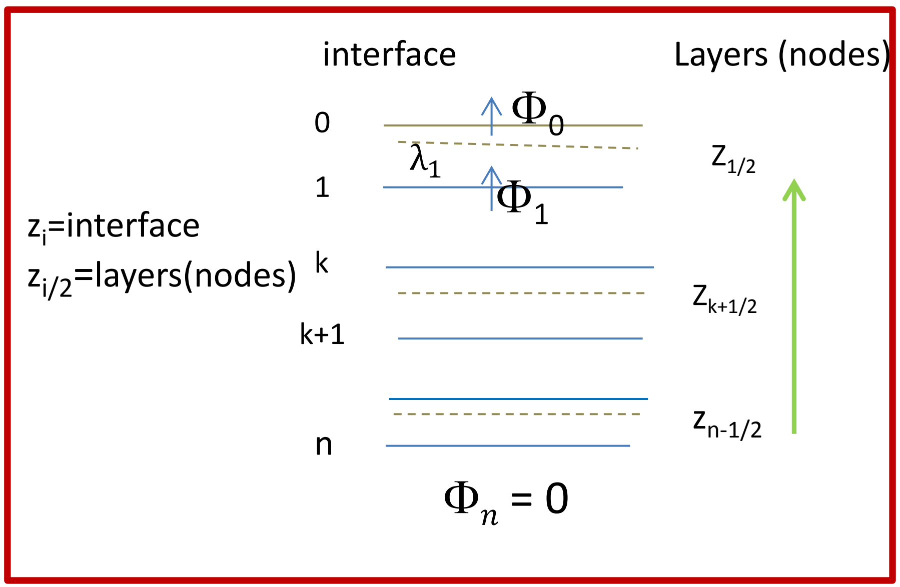
➢ Tridiagonal system as for the atmosphere (different boundary conditions)

- Heat conduction : Diffusion equation $C\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)$

$$\Phi_T = -\lambda\frac{\partial T}{\partial z} \ ;$$

- Top: Continuity between sub-surface and atmosphere + vertical discretization $\Phi_o = \text{Rad} + \sum F^{\downarrow}(T_S^t) - \varepsilon\sigma(T_S^t)^4$

$$C_{p_{1/2}}^t \frac{T_{1/2}^t - T_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{T_{3/2}^t - T_{1/2}^t}{z_{3/2} - z_{1/2}}\right] + Rad + \sum F^{\downarrow}(T_S^t) - \varepsilon\sigma(T_S^t)^4$$



- Intermediate layers

$$C_{p_{k+1/2}}^t \frac{T_{k+1/2}^t - T_{k+1/2}}{\delta t} = \frac{1}{z_{k+1} - z_k}\left[\lambda_{k+1}\frac{T_{k+3/2}^t - T_{k+1/2}^t}{z_{k+3/2} - z_{k+1/2}} - \lambda_k \frac{T_{k+1/2}^t - T_{k-1/2}^t}{z_{k+1/2} - z_{k-1/2}}\right]$$

- **Bottom** : $\Phi = 0$ $\quad C_{p_{n-1/2}}^t \frac{T_{n-1/2}^t - T_{n-1/2}}{\delta t} = \frac{1}{z_N - z_{N-1}}\left[-\lambda_{n-1}\frac{T_{n-1/2}^t - T_{n-3/2}^t}{z_{n-1/2} - z_{n-3/2}}\right]$

- Heat conduction : Diffusion equation

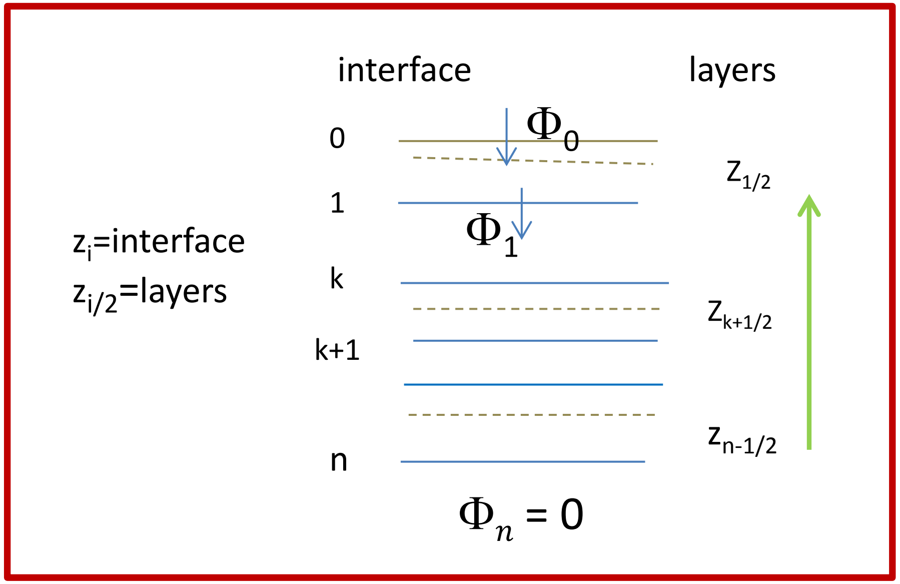
$$\Phi_T = -\lambda \frac{\partial T}{\partial z} \;;$$

$$\frac{\partial T}{\partial t} = -\frac{1}{C}\frac{\partial \Phi_T}{\partial z}$$

We obtain by recurrence (same as for atmosphere)

- **Top: Continuity between sub-surface and atmosphere**

$$\mathrm{Cp}_{1/2}\frac{\mathrm{T}^{t}_{1/2}-\mathrm{T}_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{\mathrm{T}^{t}_{3/2}-\mathrm{T}^{t}_{1/2}}{z_{3/2}-z_{1/2}}\right] + Rad + \sum \mathrm{F}^{\downarrow}(\mathrm{T}^{t}_{S}) - \varepsilon\sigma(\mathrm{T}^{t}_{S})^4 \qquad \mathrm{T}^{t}_{3/2} = \alpha^{t}_{1}\,\mathrm{T}^{t}_{\frac{1}{2}} + \beta^{t}_{1}$$



- Intermediate layers

$$\mathrm{T}^{t}_{k+1/2} = \alpha^{t}_{k}\,\mathrm{T}^{t}_{k-1/2} + \beta^{t}_{k}$$

At t, α_k and β_κ depend on $T_{k1/2}$ at the previous time step they can be computed with a recurrence relation from one layer to the other.

- **Bottom** : $\Phi_n = 0$

$$\mathrm{T}^{t}_{n-1/2} = \alpha^{t}_{n-1}\,\mathrm{T}^{t}_{n-\frac{3}{2}} + \beta^{t}_{n-1}$$

- Heat conduction : Diffusion equation

We obtain an inner relation

$$\Phi_T = -\lambda \frac{\partial T}{\partial z} \quad ; \quad \frac{\partial T}{\partial t} = -\frac{1}{C}\frac{\partial \Phi_T}{\partial z}$$

- Top: Continuity between sub-surface and atmosphere

(1) $$C^t_{p_{1/2}} \frac{T^t_{1/2} - T_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{T^t_{3/2} - T^t_{1/2}}{z_{3/2} - z_{1/2}}\right] + \sum F^{\downarrow}(T^t_S) - \varepsilon\sigma(T^t_S)^4$$

(2) $$T^t_{3/2} = \alpha^t_1 \, T^t_{\frac{1}{2}} + \beta^t_1$$

- Heat conduction : Diffusion equation

We obtain by recurrence:

$$\Phi_T = -\lambda \frac{\partial T}{\partial z} \;;\; \frac{\partial T}{\partial t} = -\frac{1}{C}\frac{\partial \Phi_T}{\partial z}$$

- Top: Continuity between sub-surface and atmosphere

(1) $$C_{p_{1/2}} \frac{T^t_{1/2} - T_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{T^t_{3/2} - T^t_{1/2}}{z_{3/2} - z_{1/2}}\right] + Rad + \sum F^{\downarrow}(T^t_S) - \varepsilon\sigma(T^t_S)^4$$

(2) $$T^t_{3/2} = \alpha^t_1 \, T^t_{\frac{1}{2}} + \beta^t_1$$

$$C^* \frac{T^t_{1/2} - T_{1/2}}{\delta t} = G^* + Rad + \sum F^{\downarrow}(T^t_S) - \varepsilon\sigma(T^t_S)^4$$

Ts : linearly extrapolated from $T_{3/2}$ and $T_{1/2}$

At t, α_k and β_κ depend on $T_{k1/2}$ at the previous time step they can be computed with a recurrence relation from one layer to the other.

$$C'^* \frac{T^t_S - T_S}{\delta t} = G'^* + Rad + \sum F^{\downarrow}(T^t_S) - \varepsilon\sigma(T^t_S)^4$$

Hourdin 1993 (thèse)
Wang, Cheruy, Dufresne 2016 GMD

SOLVING FOR T_s

$$C' * \frac{T_S^t - T_S}{\delta t} = G' * \underbrace{+ Rad + \sum F^{\downarrow}(T_S^t) - \varepsilon\sigma(T_S^t)^4}$$

$F^1{}_X$ = Bulk formula = $\rho\, C_d{}^x |V|\, (X_1 - T_s) = K_1\, (X_1 - T_s)$

$F^1{}_X = \frac{X_1 - A_1}{B_1 \delta t} = M_1 + N_1\, T_s$

$M_1 = \frac{K_1 A_1}{1 - \delta t\, K_1 B_1}$ $\qquad \pi = (P_0/P_1)^k$

$N_1 = -\frac{\pi K_1 A_1}{1 - \delta t\, K_1 B_1}$

Solving the tridiagonal system

At the bottom of the boundary layer $X_2 = C_2^X + D_2^X X_1$

$$\left(\delta P_1 + R_2^X\right) X_1 = \delta P_1 \; X_1^0 + R_2^X \; X_2 - g\delta t F_1^X$$

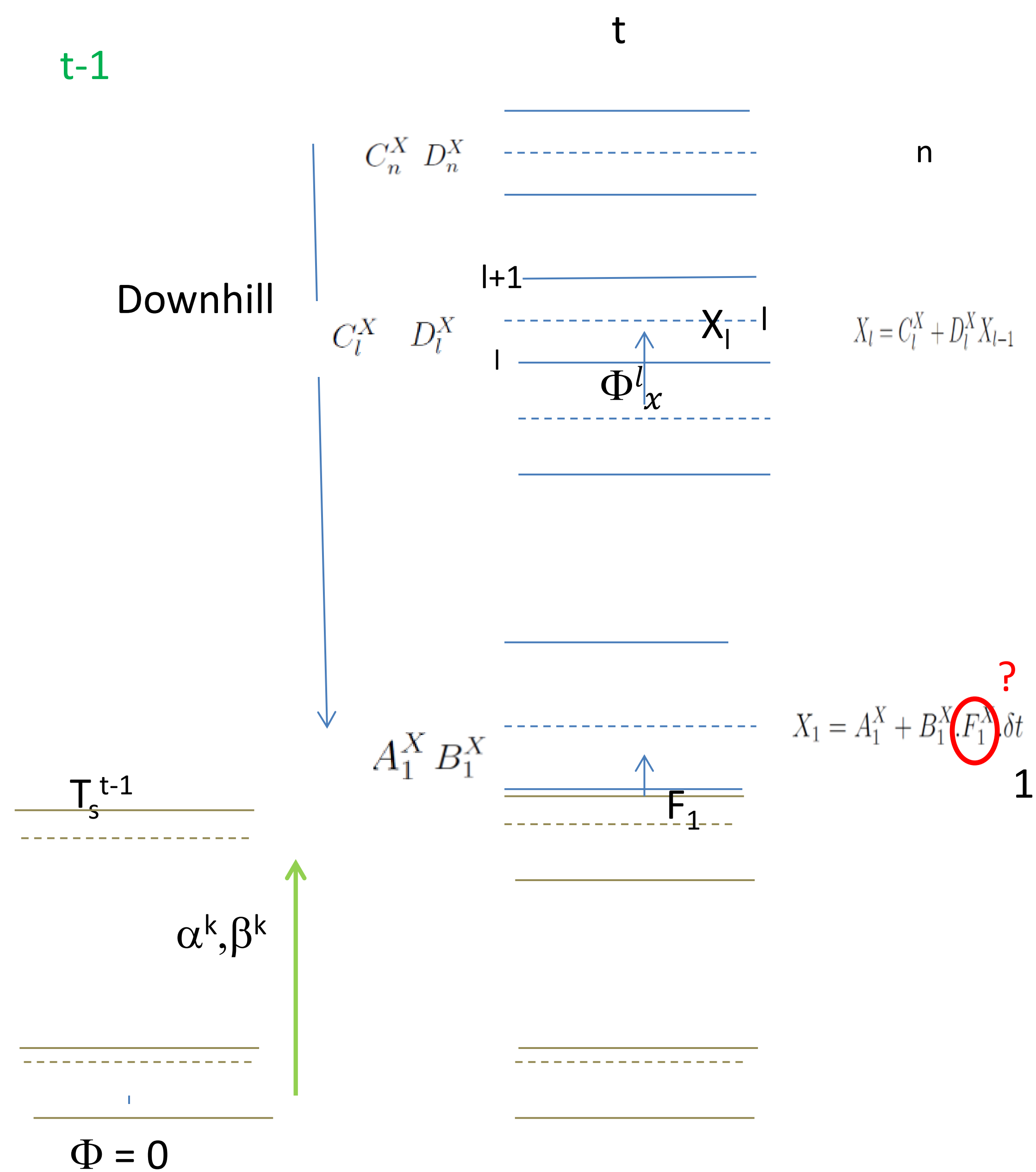
replacing X_2 in the equation above:

$$\boxed{X_1 = A_1^X + B_1^X . F_1^X . \delta t}$$

with

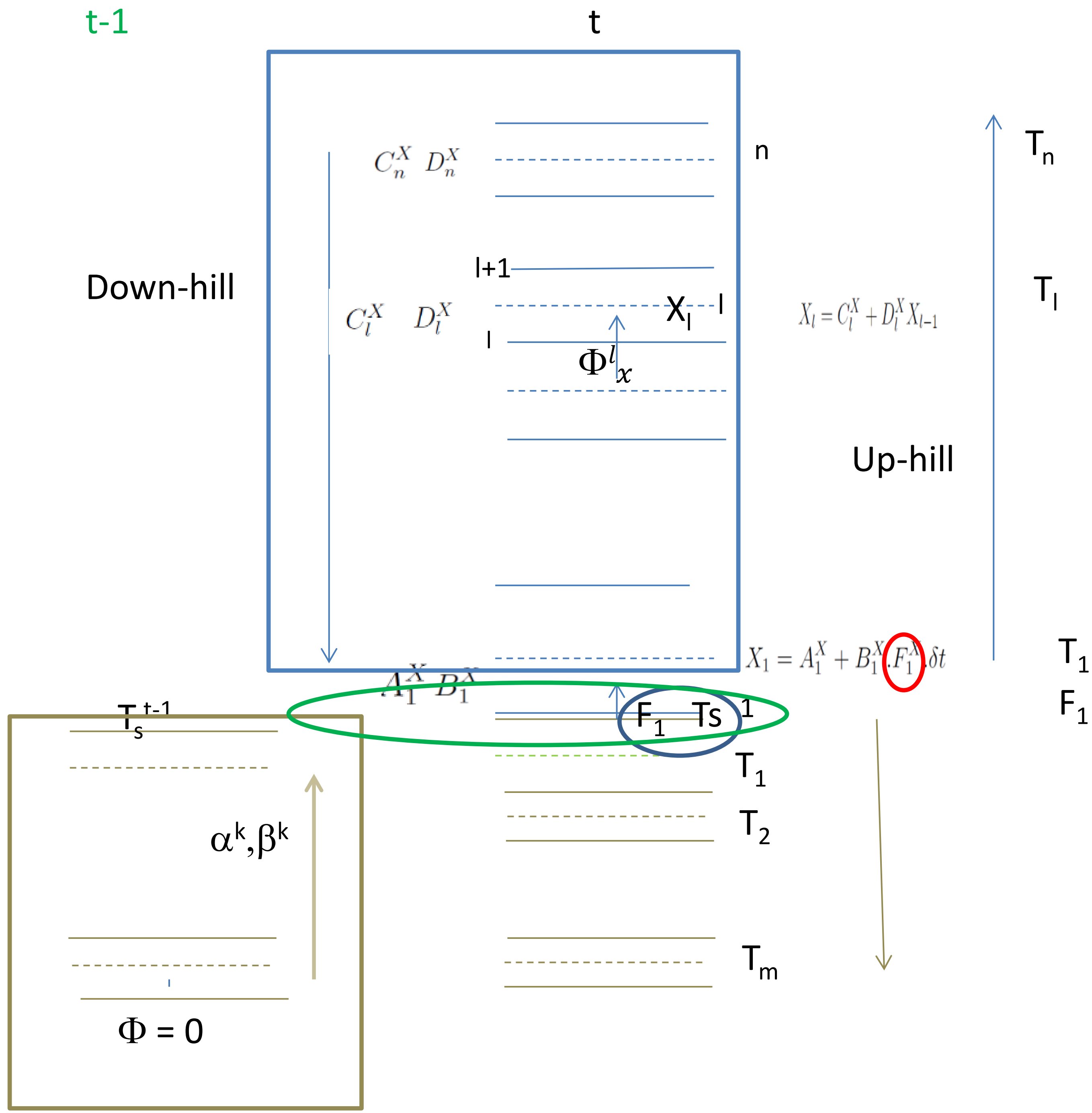
$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X (1 - D_2^X)}$$

$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$



At t α_k and β_κ depend on T_k at the previous time step and on the underlying layers :
They can be pre-computed

Hourdin 1993



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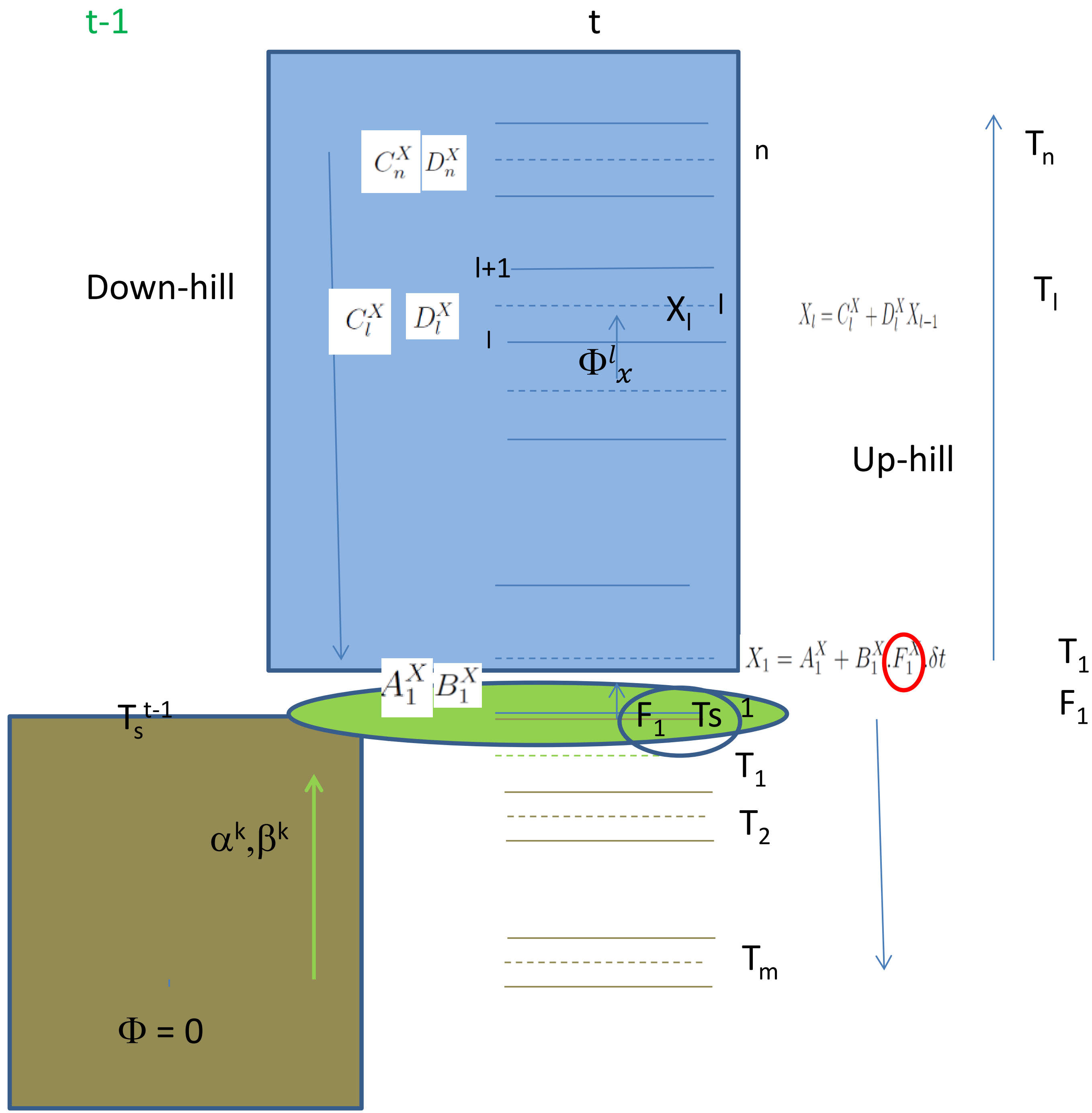


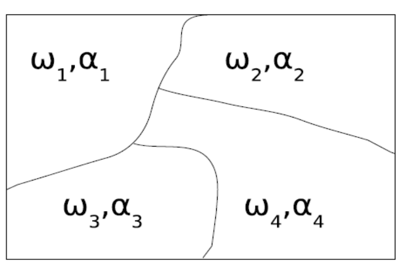
At t α_k and β_κ depend on T_k at the previous time step and on the underlying layers :
They can be pre-computed

Coupling between atmospheric column(s) and sub-surfaces

Each grid cell is divided into several sub-areas or ``sub-surfaces" of fractions ω_i

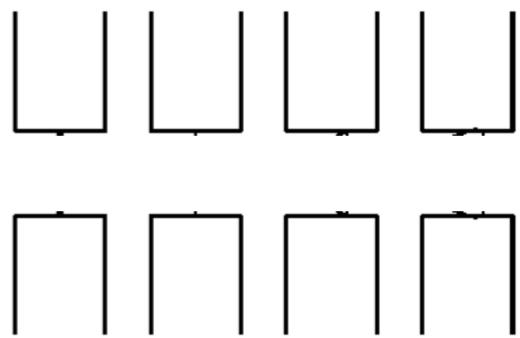
Sub-surfaces

$$\sum_i \omega_i = 1$$



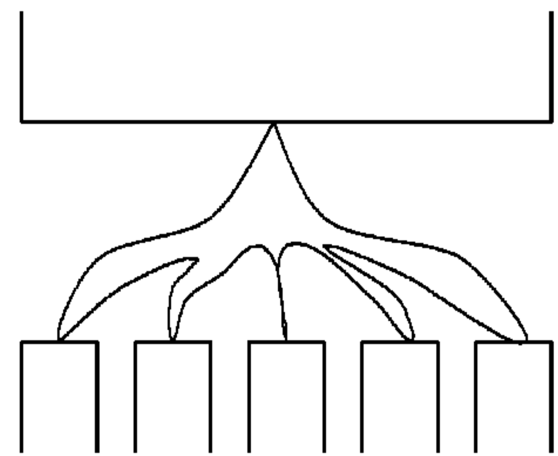
Turbulent flux

One PBL over **each** sub-surface



Radiative flux

One column **covers all** the sub-surface



Each sub surface has to compute *F_1* using variables *X_1*, *A_1* and *B_1*

The boundary layer tendencies in the atmosphere are mixed between sub-columns (equivalent of averaging the surface flux)

Derivation of local sub-surface **net solar radiation** from grid average net solar radiation

The grid average net flux Ψs at surface has been computed for each grid point by the radiative code.

We want (1) to conserve energy and (2) to take into account the value of the local albedo αi of the sub-surface

We compute the downward SW radiation as

with the mean albedo $$\alpha = \sum_i \omega_i \alpha_i \qquad F_{\downarrow}^{s} = \frac{\Psi_s}{(1-\alpha)}$$

For each sub-surface i, the absorbed solar radiation reads: $$\psi_i^s = (1-\alpha_i) F_{\downarrow}^{s}$$

$$\boxed{\psi_i^s = \frac{(1-\alpha_i)}{(1-\alpha)} \bar{\Psi}^s}$$

One may verified that this procedure ensure energy conservation, i.e. $$\sum_i \omega_i \psi_i^s = \Psi_s$$

Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

The net longwave (LW) radiation $\bar{\Psi}^L$ has been computed by the radiative code for each grid cell. How to split it depending on the sub-surfaces local properties and ensuring energy conservation?

If the downward longwave flux $F_\downarrow$ is uniform within each grid, the net LW flux for a sub-surface i may be written as:

$$\psi_i^L(T_i) = \epsilon_i \left(F_\downarrow - \sigma T_i^4\right) \tag{1}$$

where T_i is the surface temperature of sub-surface i and ϵ_i its emissivity. A linearization around the mean temperature $\bar{T}$ gives:

$$\psi_i^L(T_i) \approx \epsilon_i \left(F_\downarrow - \sigma \bar{T}^4\right) - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T}) \tag{2}$$

To conserve the energy, the following relationship must be true:

$$\sum_i \omega_i \psi_i^L = \bar{\Psi}^L \tag{3}$$

Using Eq. 2 gives

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon} \left(F_\downarrow - \sigma \bar{T}^4\right) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i \left(T_i - \bar{T}\right) \tag{4}$$

where $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$ is the mean emissivity.

Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon}\left(F_\downarrow - \sigma \bar{T}^4\right) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i \left(T_i - \bar{T}\right) \tag{4}$$

where $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$ is the mean emissivity. The second term on the right hand side is zero if

$$\boxed{\bar{T} = \frac{\sum_i \omega_i \epsilon_i T_i}{\bar{\epsilon}}} \tag{5}$$

To ensure energy conservation, we need in addition to verify:

$$\bar{\Psi}^L = \bar{\epsilon}\left(F_\downarrow - \sigma \bar{T}^4\right) \tag{6}$$

Which is consistent with the definition of the net LW flux at the surface. We rewrite now Eq. 2 as:

$$\boxed{\psi_i^L(T_i) \quad \approx \quad \frac{\epsilon_i}{\bar{\epsilon}} \bar{\Psi}^L - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T})} \tag{7}$$

Due to radiative code limitation, in LMDZ, we always must have $\varepsilon_i = 1$

Call tree

In subroutine PHYSIQ

loop over time steps

CALL change_srf_frac : Update fraction of the sub-surfaces (pctsrf)

....

CALL pbl_surface Main subroutine for the interface with surface

Calculate net radiation at sub-surface

Loop over the sub-surfaces nsrf

Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface in not zero)

CALL cdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)

CALL coef_diff_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)

CALL climb_hq_down downhill for enthalpie H and humidity Q

CALL climb_wind_down downhill for wind (U and V)

CALL surface models for the various surface types: **surf_land, surf_landice, surf_ocean or surf_seaice**. **Each surface model computes:**

- evaporation, latent heat flux, sensible heat flux
- surface temperature, albedo (emissivity), roughness lengths

CALL climb_hq_up : compute new values of enthalpie H and humidity Q

CALL climb_wind_up : compute new values of wind (U and V)

Uncompress variables : (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)

Cumulate in global variables after weighting by sub-surface fractions

Surface diagnostics : (T, q, wind are evaluated at a reference level (2m) owing to an interpolation scheme based on the MO laws).

End Loop over the sub-surfaces

Calculate the mean values over all sub-surfaces for some variables

End pbl-surface

About the surface models: land, ocean, sea-ice , land-ice

For land : simplified land surface model, hydrology= bucket or “beta clim”,
constant thermal inertia (soil /snow) , albedo and rugosity from a file.
or SVAT model (ORCHIDEE)

For ocean: Forced, fully coupled (with NEMO), coupled with a slab-ocean

For sea ice depends on the coupling with the ocean (forced, coupled, slab)

For land-ice : snow properties calculated with sisvat if ok_snow= T.
otherwise simplified (as for land + simplified snow prop., rugo, albedo)

About the values interpolated at a reference level near the surface (e.g. 2m)

Principle: Constant flux in the surface layer and similarity laws: Non dimensional vertical gradient of horizontal wind, potential temperature, specific humidity are assumed to be universal function of a stability parameter z/L (L= Monin-Obukhov law) or of the Richardson Number.

$$\frac{kz}{u_*}\frac{\partial u}{\partial z} = \phi_M\left(\frac{z}{L}\right) \; ; \; \frac{kz}{\theta_*}\frac{\partial \theta}{\partial z} = \phi_H\left(\frac{z}{L}\right) \; ; \; \frac{kz}{q_*}\frac{\partial q}{\partial z} = \phi_q\left(\frac{z}{L}\right)$$

integrating these equations gives :

$$\frac{ku}{u_*} = \ln\left(\frac{z}{z_0}\right) - \Psi_M\left(\frac{z}{L}\right) + \Psi_M\left(\frac{z_0}{L}\right)$$

$$\frac{k(\theta-\theta_0)}{\theta_*} = \ln\left(\frac{z}{z_0}\right) - \Psi_H\left(\frac{z}{L}\right) + \Psi_H\left(\frac{z_{0H}}{L}\right)$$

Louis shows that one can use the Richardson (bulk) number instead of the Monin Obukhov length

$$\frac{ku}{u_*} = \frac{\ln(z/z_0)}{F_M^{1/2}\left(Ri^b, \frac{z}{z_0}\right)}$$

II $$\frac{k(\theta-\theta_0)}{\theta_*} = \frac{\ln\left(\frac{z}{z_0}\right)}{F_H\left(Ri^b, \frac{z}{z_0}\right)} F_M^{1/2}\left(Ri^b, \frac{z}{z_0}\right)$$

Louis shows that one can use the Richardson (bulk) number instead of the monin obukhov length

$$\frac{k\,u}{u_*} = \frac{\ln(z/z_0)}{F_M^{1/2}\left(R_i^b, \frac{z}{z_0}\right)}$$

$$\text{II} \quad \frac{k(\theta-\theta_0)}{\theta_*} = \frac{\ln\left(\frac{z}{z_0}\right)}{F_H\left(R_i^b, \frac{z}{z_0}\right)} F_M^{1/2}\left(R_i^b, \frac{z}{z_0}\right)$$

if on writes II for 2 levels (first atm level and reference level)

$$\frac{\theta_{ref}-\theta_0}{\theta_1-\theta_0} = \frac{\ln\left(\frac{z_{ref}}{z_0}\right)}{\ln\left(\frac{z_1}{z_0}\right)} \cdot \frac{F_M^{1/2}\left(R_i^{ref}, \frac{z_{ref}}{z_0}\right)}{F_M^{1/2}\left(R_i^1, \frac{z_1}{z_0}\right)} \cdot \frac{F_H\left(R_i^1, \frac{z_1}{z_0}\right)}{F_H\left(R_i^{ref}, \frac{z_{ref}}{z_0}\right)}$$

$\theta_0 = T_S$

$\longrightarrow$ evaluate $\theta_{ref} = f(\theta_0, \theta_1, \text{stability}, z_1, z_{ref}, R_i^1, R_i^R)$

$$\frac{q_{ref}-q_0}{q_1-q_0} = \frac{\ln\left(\frac{z_{ref}}{z_0}\right)}{\ln\left(\frac{z_1}{z_0}\right)} \cdot \frac{F_M^{1/2}\left(R_i^{ref}, \frac{z_{ref}}{z_0}\right)}{F_M^{1/2}\left(R_i^1, \frac{z_0}{z_0}\right)} \cdot \frac{F_H\left(R_i^1, \frac{z_1}{z_0}\right)}{F_H\left(R_i^{ref}, \frac{z_{ref}}{z_0}\right)}$$

Some properties –

stability function in the stable case : $\psi_H\left(\frac{z}{L}\right) = -\frac{5}{L}$

$(L > 0)$

$$\frac{\theta - \theta_0}{\theta_*} = \frac{1}{k}\left[\ln\frac{z}{z_0} + 5\frac{z}{L}\right]$$

at 1-, $\theta_* = cst$ and does not depend on z.

$$\Rightarrow d\frac{\delta\theta}{\delta z} = \theta_* = cste = \text{function monotone of } z$$

For a stability: $\theta_S \leq \theta \leq \theta_1$

In the unstable case : $\frac{\theta - \theta_0}{\theta_*} = \ln\left(\frac{z}{z_0}\right) + 2\ln\left[\left(1 + \left(1 - 16\frac{z}{L}\right)^{2}\right]\right.$
$L' < 0$

$d\frac{\delta\theta}{\delta z}$ not always monotone –

$$F'\left(\frac{z}{L}\right) \cdot -\frac{16}{L} \cdot \frac{1}{\left(1 - 16\frac{z}{L}\right)^{1/2}\left[1 + \left(1 - 16\frac{z}{L}\right)^{1/2}\right]}$$

$$F'\left(\frac{z}{L}\right) < 0 \cdot$$

- Technical note : Description of the interface with the surface and the computation of the turbulent diffusion in LMDZ (J.L.Dufresne) http://www.lmd.jussieu.fr/~cheruy/Coupling/pbl_surface.pdf
- Thèse F. Hourdin 1993 (section 3.3.3 and annexes)
- Wang F., F. Cheruy, J.L. Dufresne, 2016: The improvement of soil thermodynamics and its effects on land surface meteorology in the IPSL climate model. Geosci. Model Dev., 9, 363–381, 2016 www.geosci-model-dev.net/9/363/2016/
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