

LMDZ

Dynamics/physics organization,
Grids,
Time stepping,
Dissipation...

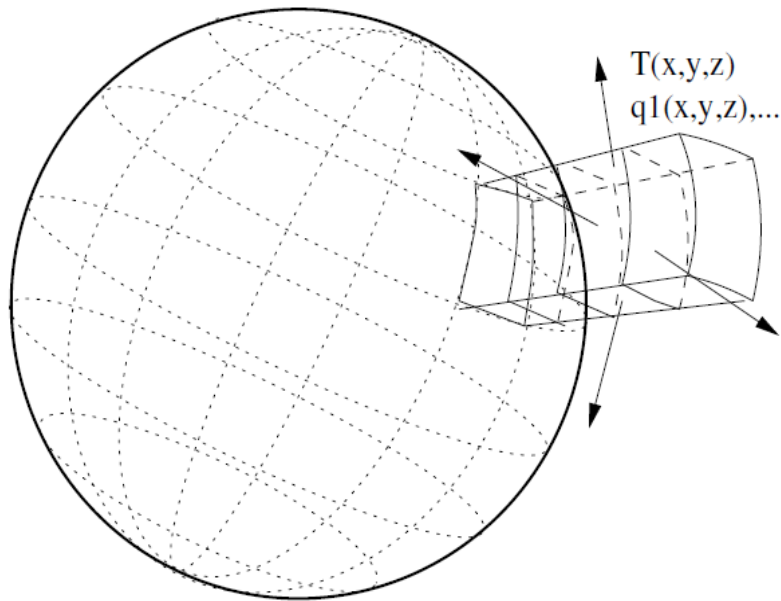
LMDZ courses, December 17 2020

Overview of course topics

- **Grids:**
 - Horizontal grids in the physics & dynamics
 - Vertical discretization
- **Time marching:**
 - Generalities about time marching schemes
 - What is used in LMDZ
 - Longitudinal polar filter
- **Lateral diffusion and sponge layer:**
 - Energy cascade
 - Illustrative example of diffusion
 - Sponge layer near model top

Grids in LMDZ

Dynamics

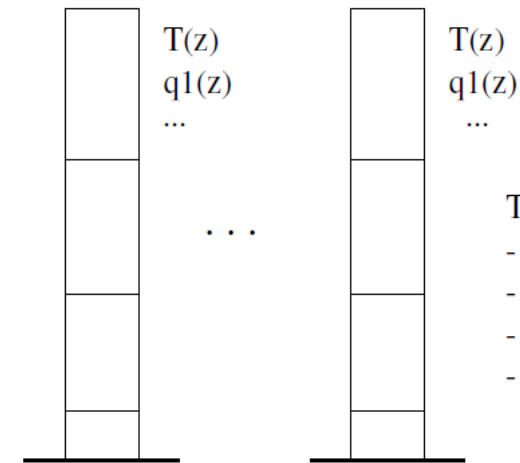


Dynamical tendencies



Physical fields

Physics



Tendencies due to
- radiative transfer
- condensation
- subgrid dynamics
- ...

Separation between physics and dynamics:

- “**dynamics**”: solving the GFD equations on the sphere; usually with the assumption of a hydrostatic balance and thin layer approximation. Valid for all terrestrial planets.
- “**physics**”: (planet-specific) local processes, local to individual atmospheric columns.

Horizontal grids in LMDZ

Grid dimensions specified
when compiling LMDZ:

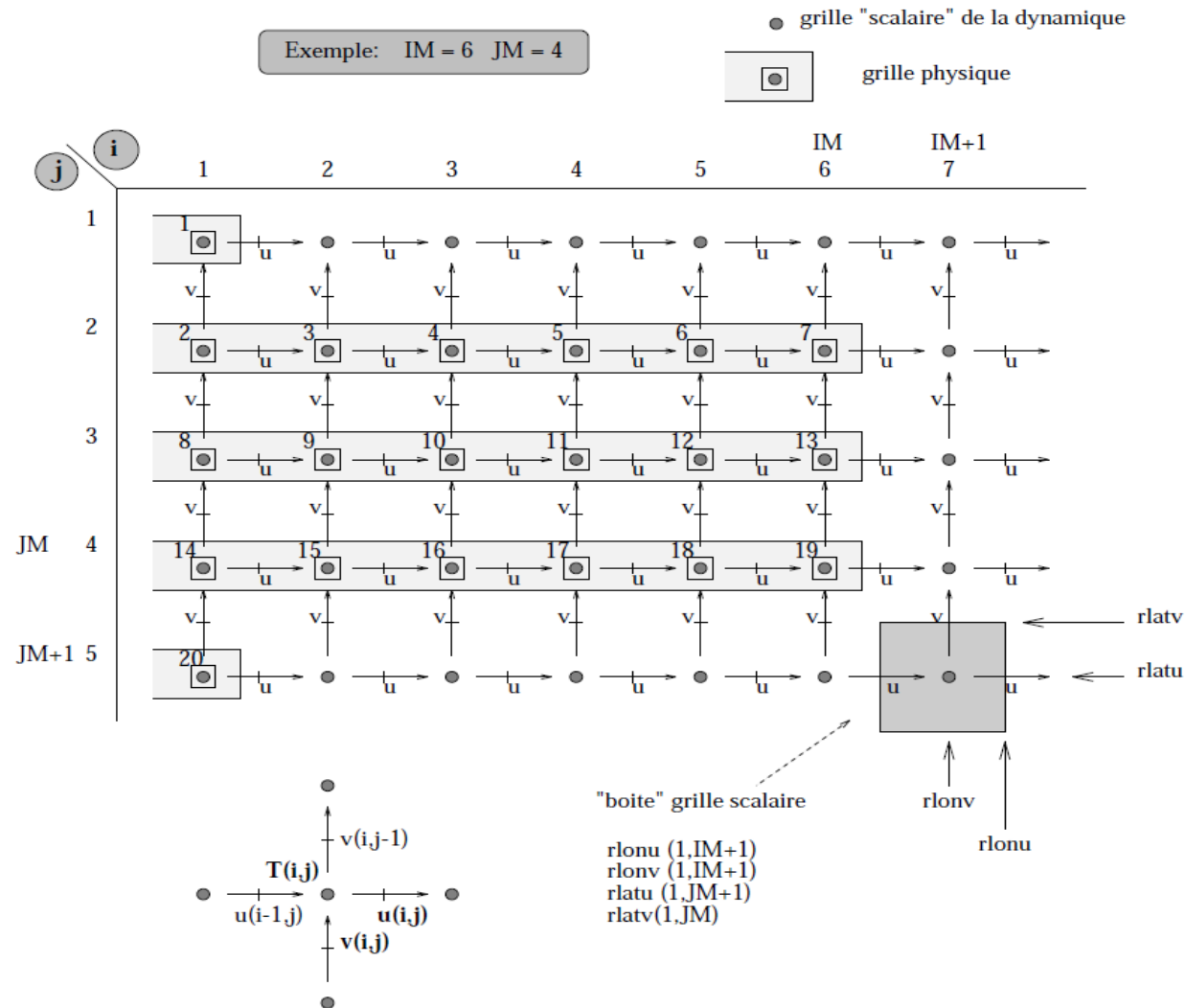
makeImdz[_fcm] -d iimxjjmxllm ...

In the dynamics:

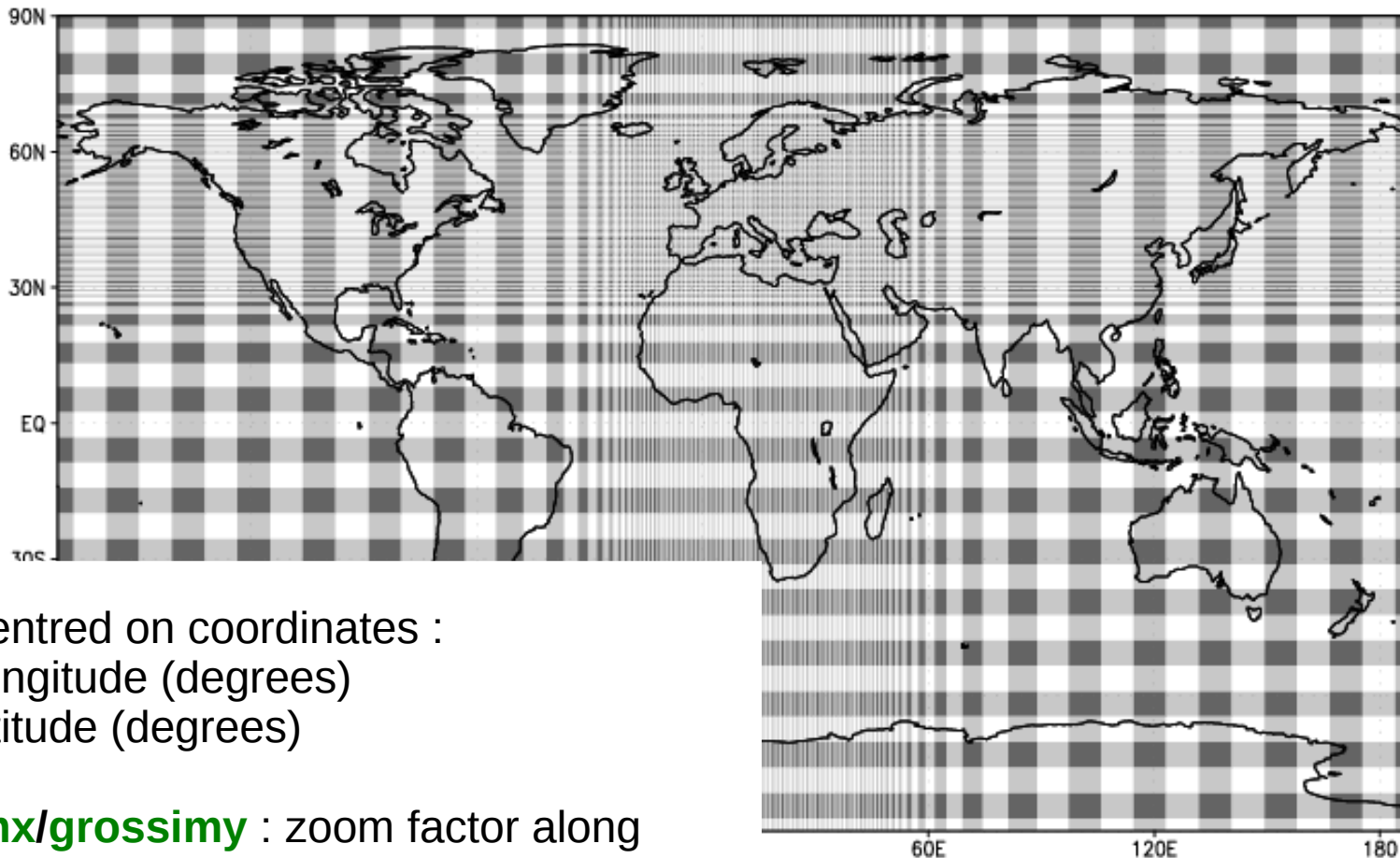
- Staggered grids, u , v and scalars (temperature, tracers) are on different meshes
- Global lonxlat grids with redundant grid points
 - at the poles
 - in longitude

In the physics:

- Collocated variables
- No global lonxlat horizontal grid, columns are labelled **using a single index** (from North Pole to South Pole)



LMDZ, Z for Zoom



Zoom centred on coordinates :

clon : longitude (degrees)

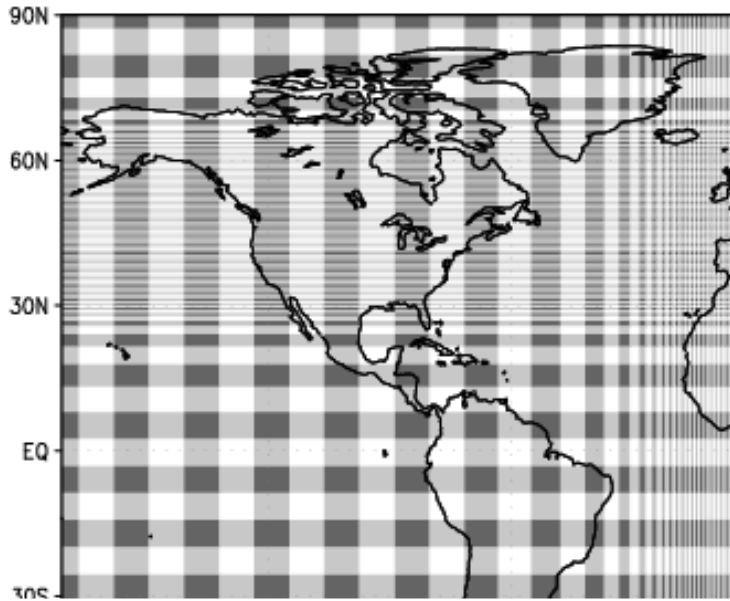
clat : latitude (degrees)

grossimx/grossimy : zoom factor along
x/y directions (i.e. lon/lat)

Computed as the ratio of the smallest mesh (i.e. in the zoom), compared to the mesh size for a global regular grid with the same total number of points.

dzoomx/dzoomy : fraction of the grid containing the zoomed area: $dzoom \times 360^\circ$ by $dzoomy \times 180^\circ$

LMD \underline{Z} , Z for Zoom



Zoom centred on coordinates :

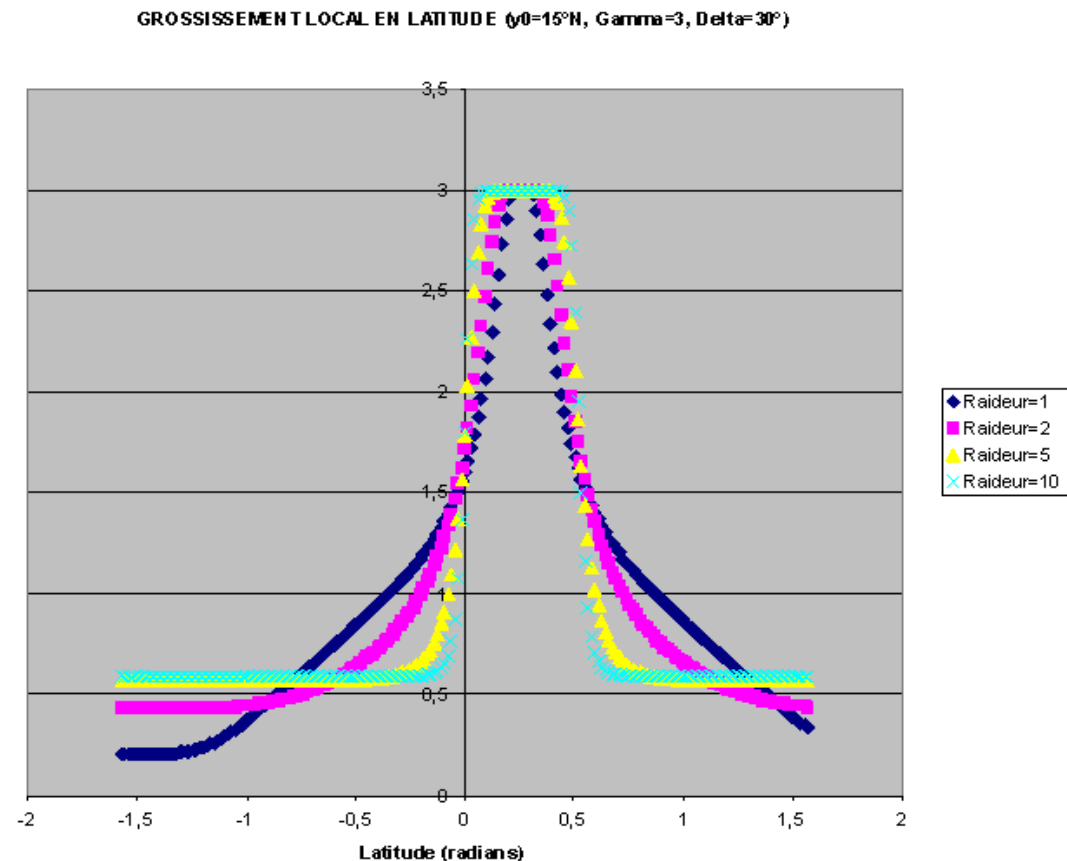
clon : longitude (degrees)

clat : latitude (degrees)

grossimx/grossimy : zoom factor along
x/y directions (i.e. lon/lat)

Computed as the ratio of the smallest mesh (i.e. in the zoom), compared to the mesh size for a global regular grid with the same total number of points.

taux/tauy : steepness of the transition between inner zoom and outer zoom meshes
(typically one tries to avoid sharp transitions; $\tau \sim 3$ is a reasonable value)



Nudging in LMDZ

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}_{GCM} + \frac{u_{analyse} - u}{\tau}$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial t}_{GCM} + \frac{v_{analyse} - v}{\tau}$$

Nudging towards
analyses or reanalyses with
given time constants

$u_{analyse}$ $v_{analyse}$

Example of nudging parameters:

ok_guide = y

guide_T = n , guide_p = n , guide_q = n

guide_u = y , guide_v = y

tau_min_u = 0.0208333 (days)

tau_max_u = 10 (days)

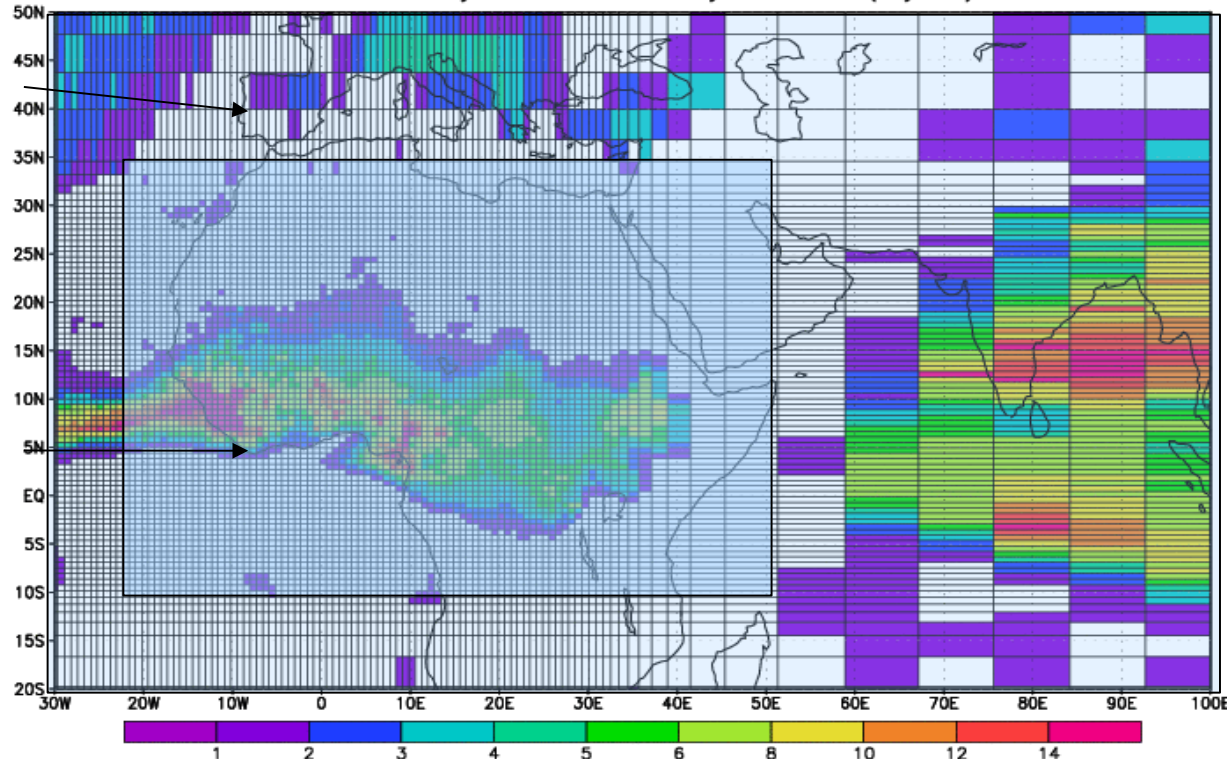
tau_min_v = 0.0208333 (days)

tau_max_v = 10 (days)

July rainfall (mm/day)
LMDZ forced by the mean annual cycle of SSTs (6 years)

Strong nudging
($\tau=30\text{min}$)

Weak to
moderate
nudging
($\tau=10\text{ days}$)



Vertical discretization in LMDZ

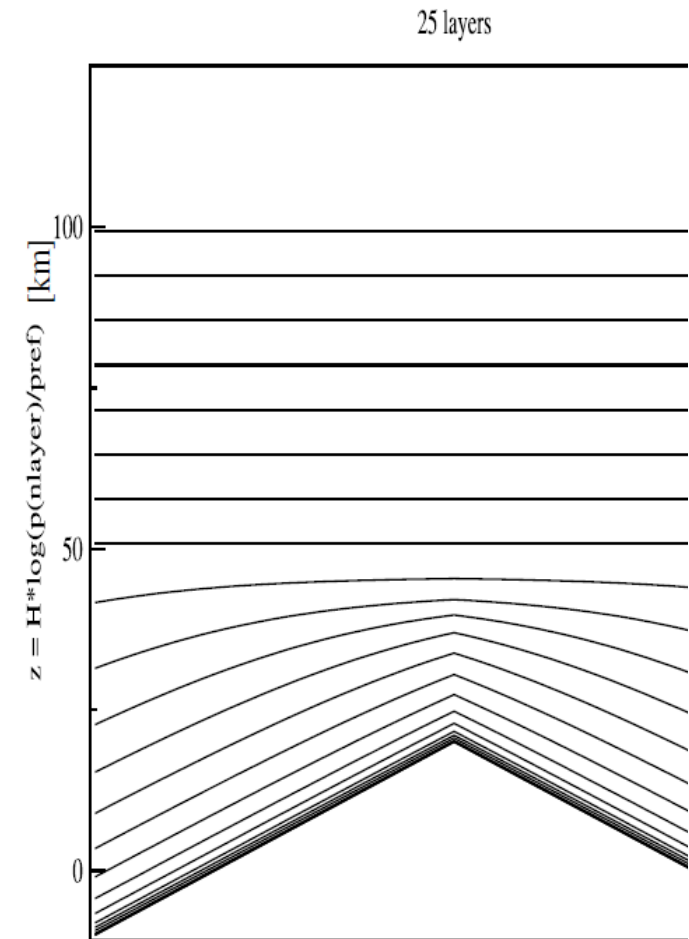
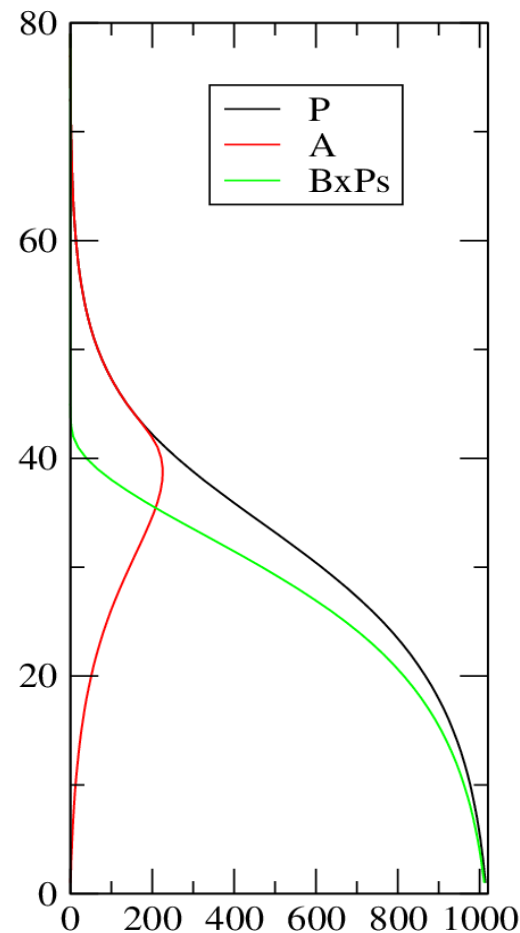
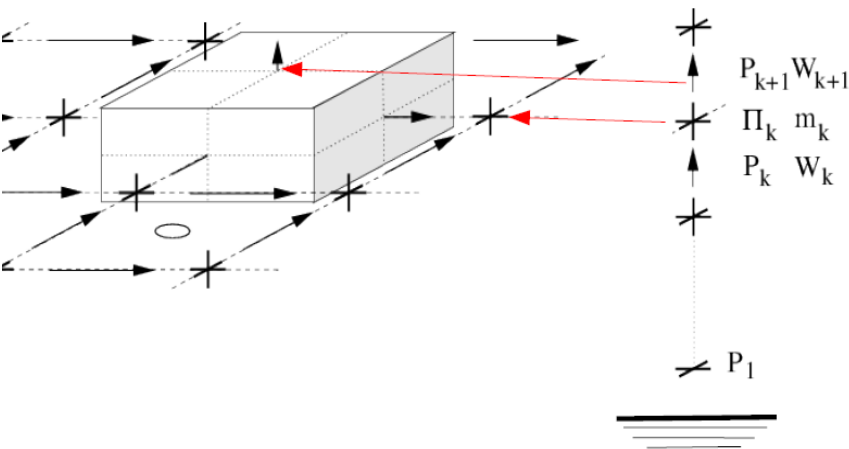
- Model levels are hybrid **sigma-pressure** levels:

$$\mathbf{P}(\text{level}, \text{time}) = \mathbf{ap}(\text{level}) + \mathbf{bp}(\text{level}) \cdot \mathbf{Ps}(\text{time})$$

hybrid coordinates $\mathbf{ap}(k)$ and $\mathbf{bp}(k)$ are fixed for a given model run

Surface pressure $\mathbf{Ps}(t)$ varies during the run

- Near the surface $\mathbf{ap} \sim 0$
 $\Rightarrow \mathbf{bp}(k) \sim P/P_s$
- At high altitudes, $\mathbf{bp} \sim 0$
 $\Rightarrow \mathbf{ap}(k) \sim P$



Vertical discretization in LMDZ

- Setting model levels via the def files (also a function of number of vertical levels) :

vert_sampling = strato_custom : customizable (via other parameters in .def file; see next slide) discretization for stratospheric extensions.

Multiple other possibilities from this “default”:

vert_sampling = strato : a default for stratospheric extensions

vert_sampling = sigma : automated generation of purely sigma levels

vert_sampling = param : load values from a “sigma.def” file

vert_sampling = tropo : a default for tropospheric simulations

vert_sampling = read : read ap() and bp() from file “hybrid.txt”

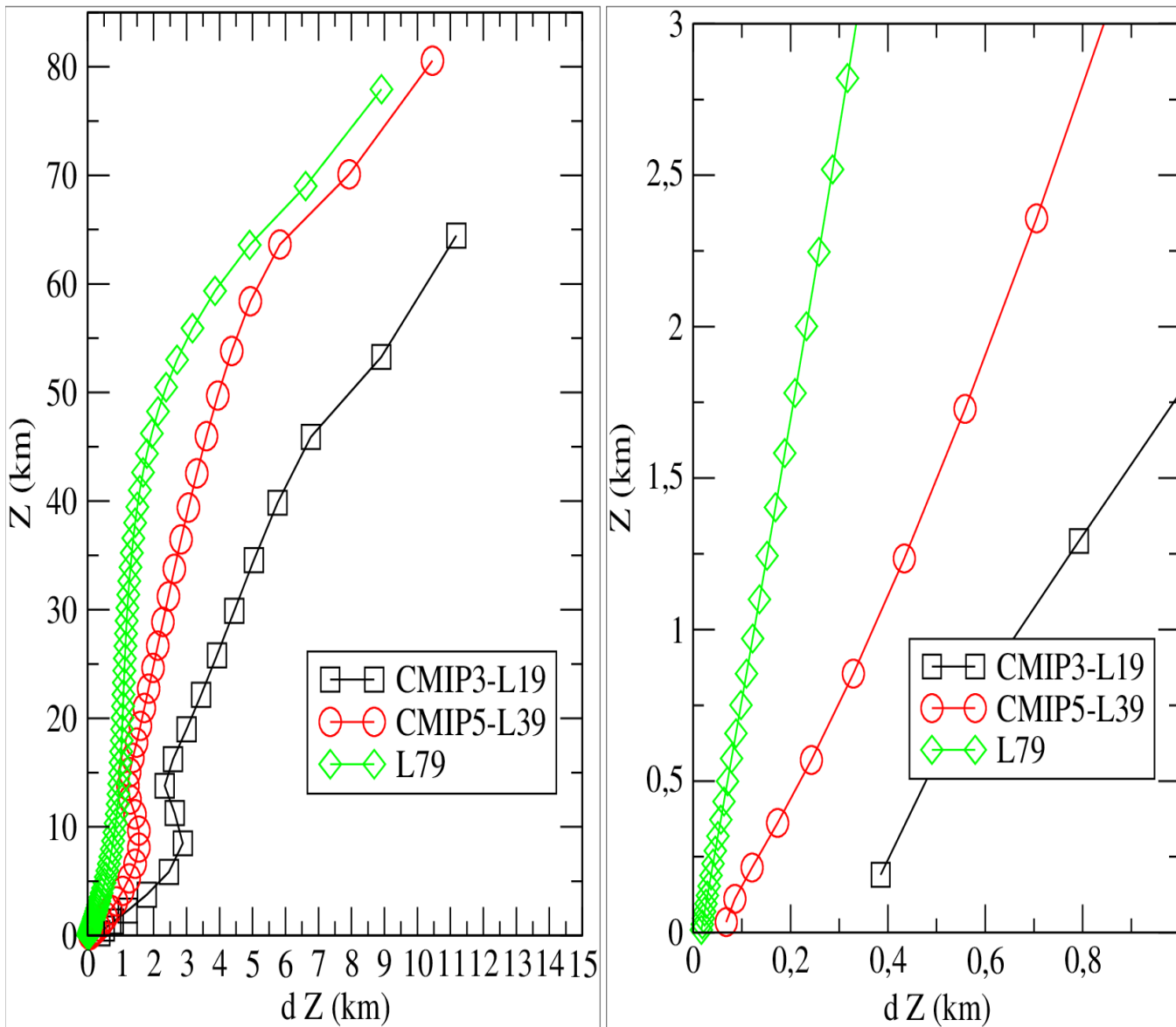
=> Typically you don't need to mess with the vertical discretization,

the default behaviour most likely matches your needs.

=> Check out routine dyn3d_common/**disvert.F90**

Vertical discretization in LMDZ

- Example of typical altitudes of L19/L39/L79 vertical layer position and thickness



“Standard” L79 settings

`vert_sampling=strato_custom`

`vert_scale=7.`
`vert_dzmin=0.017`
`vert_dzlow=1.`
`vert_z0low=8.7`
`vert_dzmid=2.`
`vert_z0mid=70.`
`vert_h_mid=20.`
`vert_dzhig=11.`
`vert_z0hig=75.`
`vert_h_hig=20.`

Questions ?

Time marching schemes

- **The big picture:** you want to solve

$$\begin{aligned}\frac{df(t)}{dt} &= R(f, t) \\ f(t=0) &= f_0\end{aligned}$$

- And it is all about using a **time marching** scheme, built on **Taylor expansion** for evaluation of the time derivative, and choosing at which **time level** $t=n\cdot dt$ the right hand side term $R[f(t),t]$ is to be evaluated

Time marching schemes

- **Explicit Euler** scheme (1st order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f, t) \simeq R(f(t_n), t_n)$$

- **Implicit Euler** scheme (1st order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f, t) \simeq R(f(t_{n+1}), t_{n+1})$$

- **Crank-Nicholson** scheme (2nd order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f, t) \simeq \frac{R(f(t_{n+1}), t_{n+1}) + R(f(t_n), t_n)}{2}$$

Time marching schemes

- **Matsuno** scheme: a predictor-corrector (Euler explicit-Euler Implicit) scheme (1st order):

$$\begin{aligned}\frac{df(t)}{dt} &\simeq \frac{f_{n+1} - f_n}{\delta t} \\ f^p(t_{n+1}) &= f(t_n) + \delta t \cdot R(f(t_n), t_n) \\ R(f, t) &\simeq R(f^p(t_{n+1}), t_{n+1})\end{aligned}$$

- **Leapfrog** scheme: use encompassing time steps to evaluate the derivative (2nd order):

$$\begin{aligned}\frac{df(t)}{dt} &\simeq \frac{f_{n+1} - f_{n-1}}{2\delta t} \\ R(f, t) &\simeq R(f(t_n), t_n)\end{aligned}$$

Time marching schemes

- Illustrative example, on a decay equation

$$\frac{dq(t)}{dt} = -\frac{1}{\tau}q(t) \longrightarrow q(t) = q_0 e^{-\frac{t}{\tau}}$$

- Building Euler explicit (E) & implicit (I) schemes:

$$\begin{aligned}\frac{dq(t)}{dt} &= -\frac{1}{\tau}q(t) \\ \Rightarrow \frac{q^{n+1} - q^n}{\delta t} &\simeq -\frac{1}{\tau}q^n \quad (\text{E.E.}) \\ \Leftrightarrow q^{n+1} - q^n &= -\frac{\delta t}{\tau}q^n \\ \Leftrightarrow q^{n+1} &= \left[1 - \frac{\delta t}{\tau}\right] q^n\end{aligned}$$

$$\begin{aligned}\frac{dq(t)}{dt} &= -\frac{1}{\tau}q(t) \\ \Rightarrow \frac{q^{n+1} - q^n}{\delta t} &\simeq -\frac{1}{\tau}q^{n+1} \quad (\text{E.I.}) \\ \Leftrightarrow q^{n+1} - q^n &= -\frac{\delta t}{\tau}q^{n+1} \\ \Leftrightarrow \frac{\tau + \delta t}{\tau}q^{n+1} &= q^n \\ \Leftrightarrow q^{n+1} &= \frac{1}{1 + \frac{\delta t}{\tau}}q^n\end{aligned}$$

Time marching schemes

- Illustrative example, on a decay equation

$$\frac{dq(t)}{dt} = -\frac{1}{\tau}q(t) \longrightarrow q(t) = q_0 e^{-\frac{t}{\tau}}$$

- Resulting integration schemes:

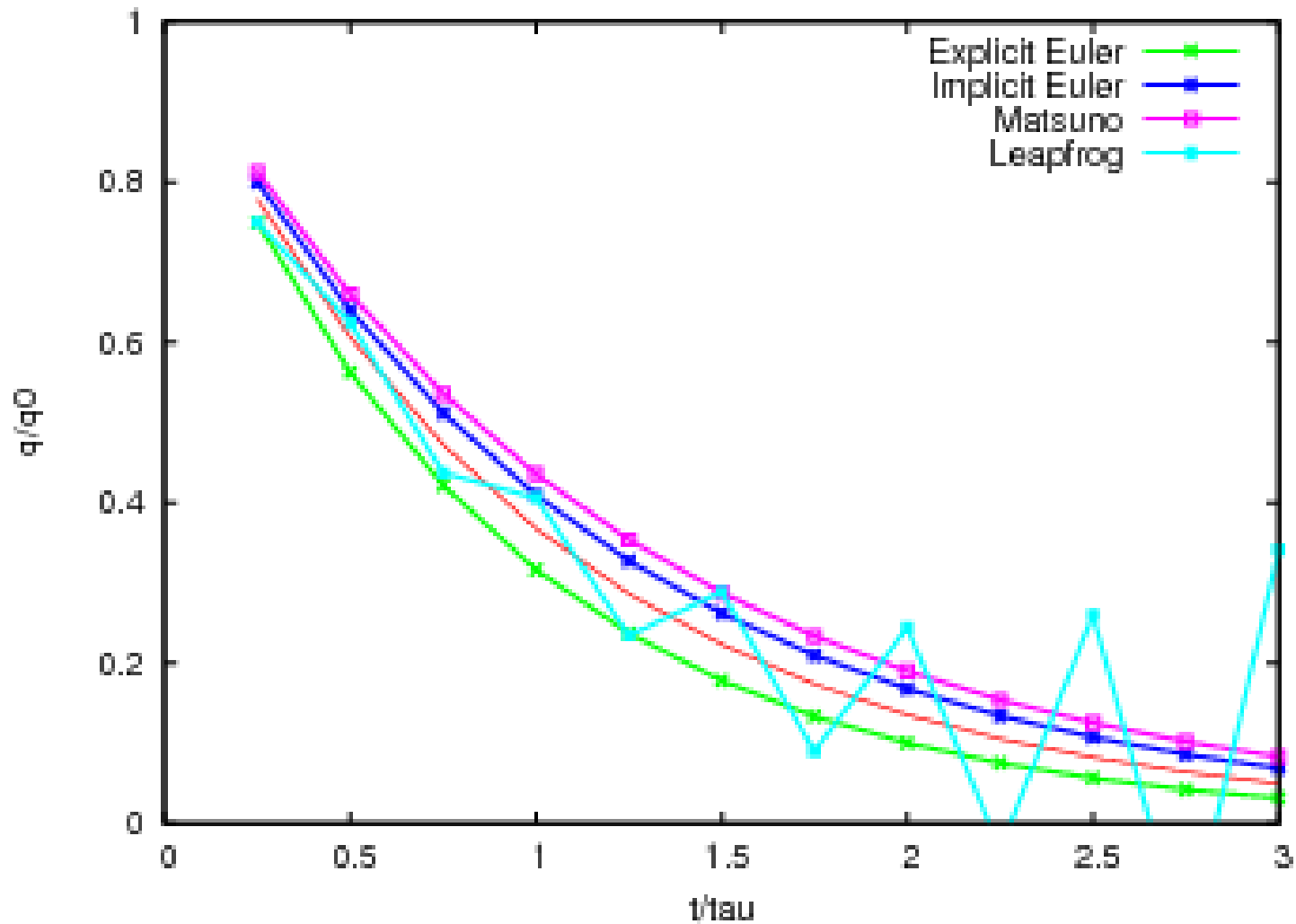
$$\text{EE : } q^{n+1} = \left[1 - \frac{\delta t}{\tau} \right] q^n$$

$$\text{EI : } q^{n+1} = \left[\frac{1}{1 + \delta t/\tau} \right] q^n$$

$$\text{CN : } q^{n+1} = \left[\frac{1 - \delta t/(2\tau)}{1 + \delta t/(2\tau)} \right] q^n$$

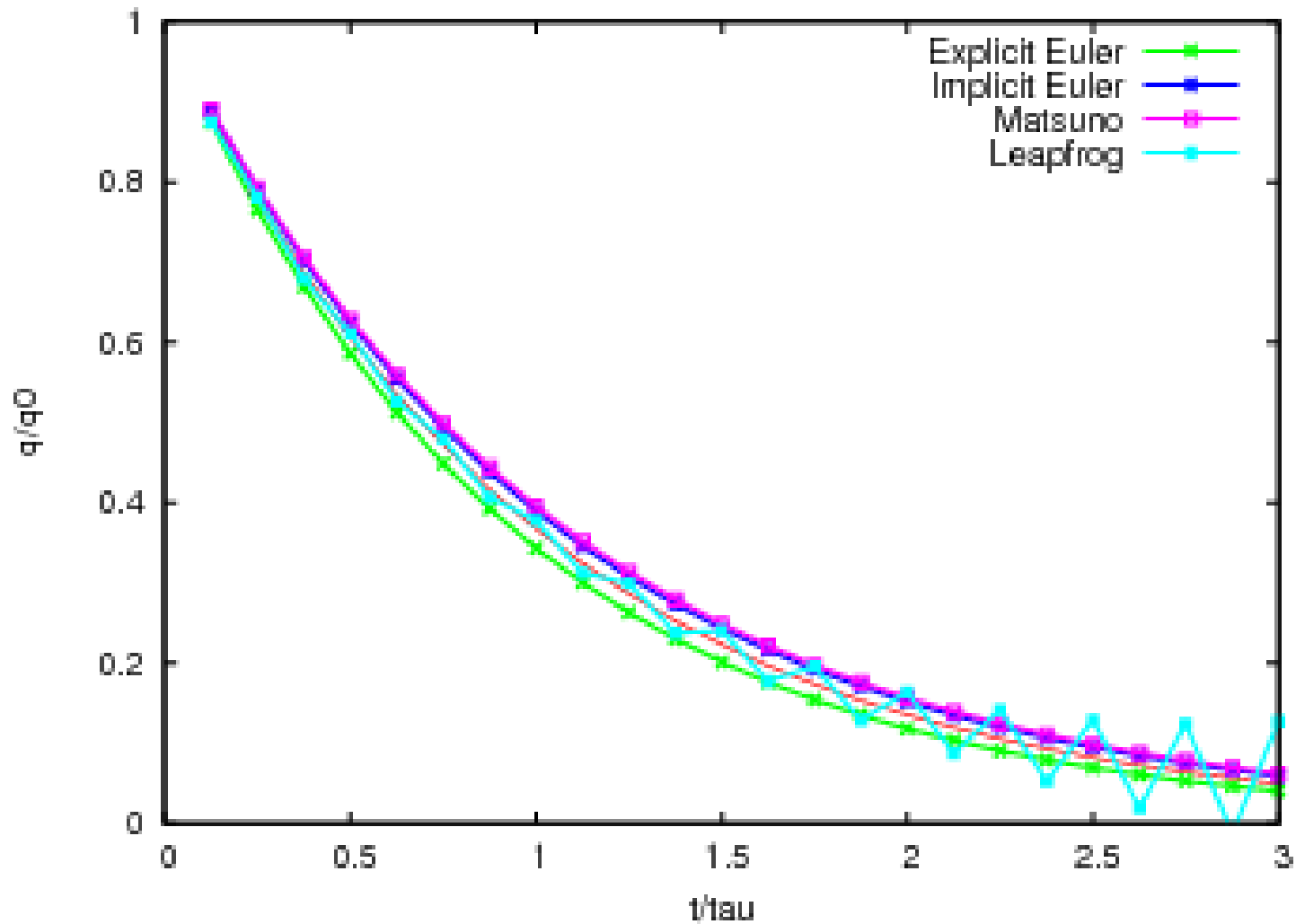
- Stability requirement (CFL) for EE : $\delta t/\tau < 2$

Time marching schemes



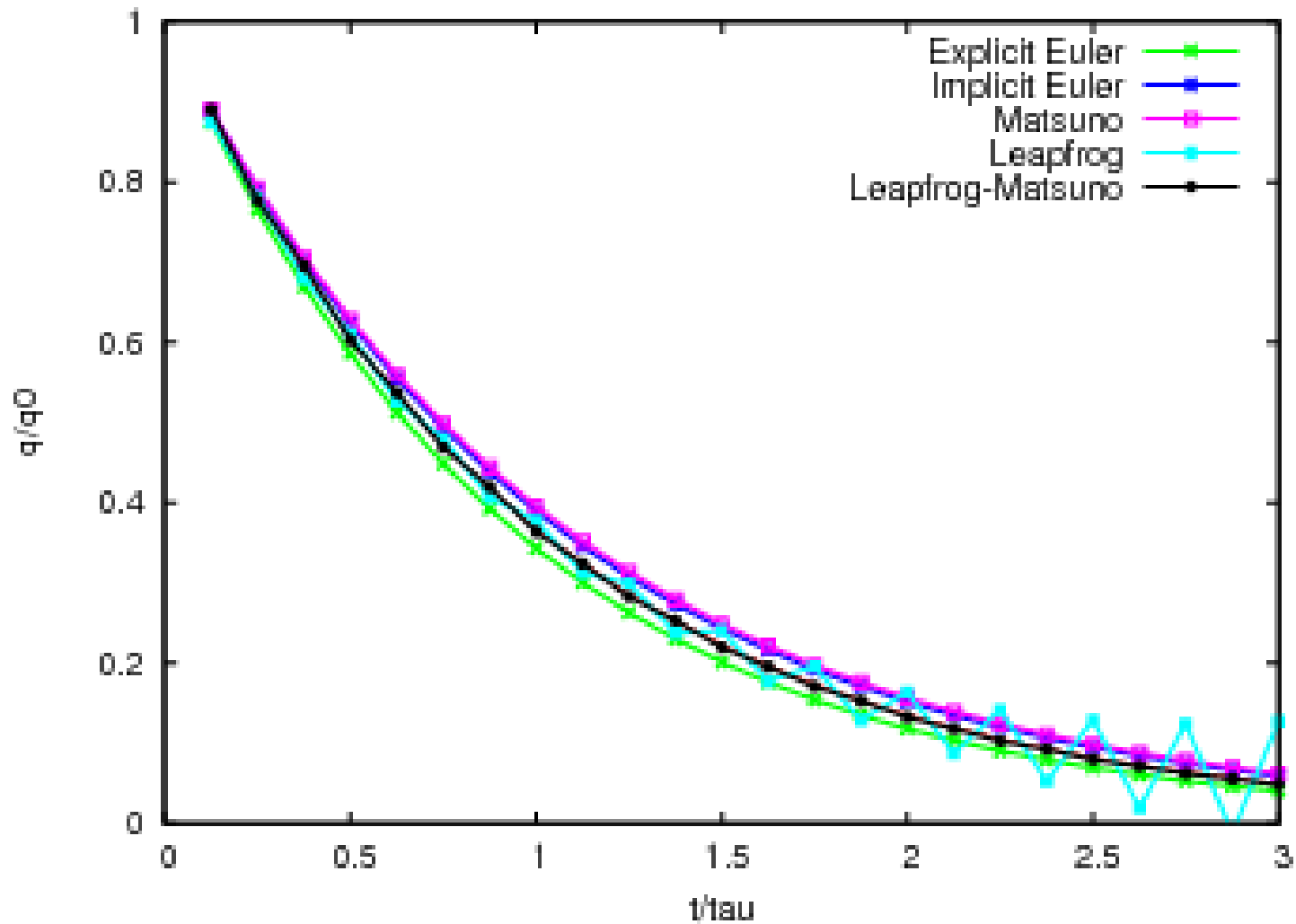
- 4 integration steps per unit of t/τ

Time marching schemes



- 8 integration steps per unit of t/τ

Time marching schemes



- 8 integration steps per unit of t/τ

Time marching in LMDZ

Time splitting between physics/dynamics/dissipation:

$$\frac{\partial \psi}{\partial t} = Dyn(\psi) + Phy(\psi) + Dissip(\psi)$$

- **Dynamics** : Leapfrog-Matsuno scheme
Using **day_step** dynamical steps per day
Leapfrog steps with a Matsuno step every **iperiod** step
- **Physics** : Explicit Euler
Every **iphysiq** dynamical steps (multiple of iperiod)
- **Dissipation**: Explicit Euler
Every **dissip_period** dynamical steps (multiple of iperiod)

Side note about explicit or implicit time marching schemes

Even when solving linear spatio-temporal boundary-value problems, e.g.:

$$\frac{dA}{dt} = \kappa \frac{\partial^2 A}{\partial x^2}$$

The **explicit Euler** approach leads to a straightforward expression for grid point values (but with stability constraints) :

$$\frac{A_i^{k+1} - A_i^k}{\delta t} = \frac{\kappa}{h^2} [A_{i-1}^k - 2A_i^k + A_{i+1}^k]$$

Whereas the **implicit Euler** approach leads to a (tridiagonal) system of equations to solve:

$$\frac{A_i^{k+1} - A_i^k}{\delta t} = \frac{\kappa}{h^2} [A_{i-1}^{k+1} - 2A_i^{k+1} + A_{i+1}^{k+1}]$$

=> **requires more computations**, but may be necessary if time-stepping constraints require using large time steps.

Side note about tridiagonal system solving

When needing to solve a tridiagonal system of the form:

$T.x=y$, T tridiagonal matrix, x & y vectors

Rather than invert T (costly!) to generate T^{-1} (dense matrix) and compute $x=T^{-1}.y$ (matrix-vector product)

Use the LU decomposition (Gaussian elimination) of T to split the problem into two very simple sub-problems:

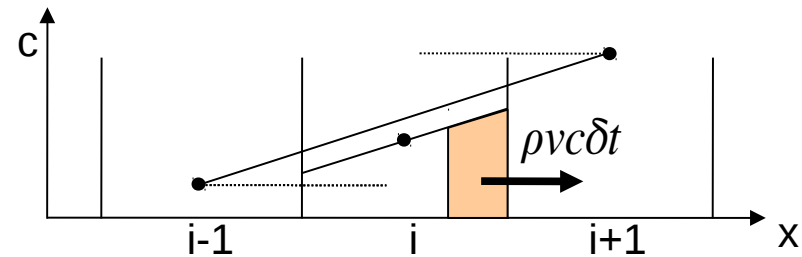
- 1) $L.U=T$, L and U are bidiagonal (lower/upper) matrices
- 2) Solve $L.z=y$ for vector z (forward substitution step)
- 3) Solve $U.x=z$ for vector x (backward substitution step)

Tracer advection in LMDZ

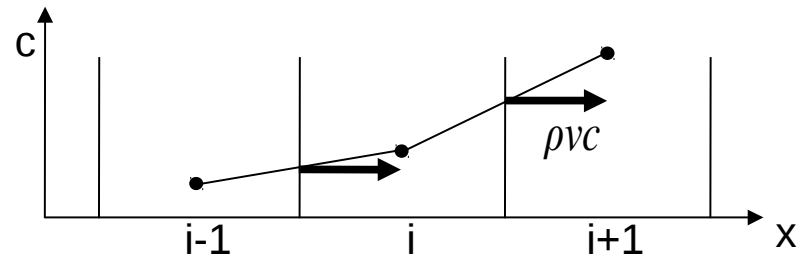
Use of the **Van Leer I scheme** (1977), a second order finite volume scheme with slope limiters (e.g. MUSCL, MINMOD) (Hourdin et Armengaud, 1999).

Guaranties of fundamental physical properties of transport :
conservation of the total quantity,
positivity, monotony, non
amplification of extrema, weak
numerical diffusion

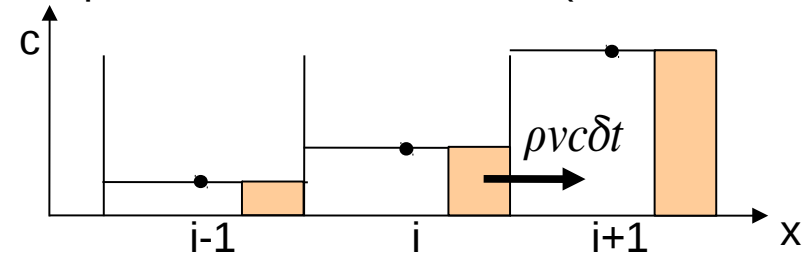
Scheme I by Van Leer (1977)



Centered finite differences (second order)



Upwind first order scheme (Godunov, 1952)



- **CFL requirement**, for an advection velocity U_{\max} :
 $U_{\max} \cdot (dt/dx) = cte$, with $cte \sim 0(1)$

Tracer advection in LMDZ

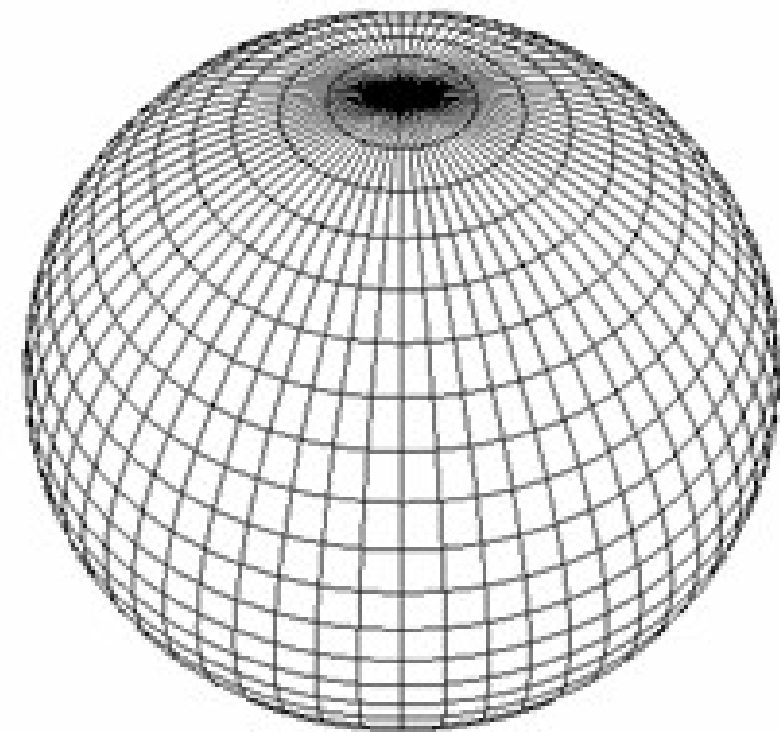
- In practice: Tracer names and advection schemes are set in the `traceur.def` file. e.g.:

5	<- Total number of tracers
14 14 H2Ov	<- Tracer advection scheme and name
10 10 H2OI	<- Tracer advection scheme and name
10 10 H2Oi	<- ...
10 10 RN	
10 10 PB	

- Scheme “10” : Van Leer scheme
- Scheme “14” : Specific modified scheme for water vapor
- Other (experimental) schemes are coded; see [dyn3d/advtrac.F90](#)

Questions ?

Longitudinal polar filter



- A lon-lat grid implies that the **meshes tighten dramatically** as the pole is approached.
- **CFL conditions** there would dictate using an extremely small time step for the time marching scheme.
- **Longitudinal** (Fourier) **filtering**, removing high spatial frequencies, is used to enforce that resolved features are at the level of those at $\sim 60^\circ$

Energy spectra and lateral dissipation

- Observations (Nastrom & Gage 1985, Lindborg 1999) collected over length scales from a few to thousands of km display a **characteristic energy cascade** (from Skamarock, 2004).

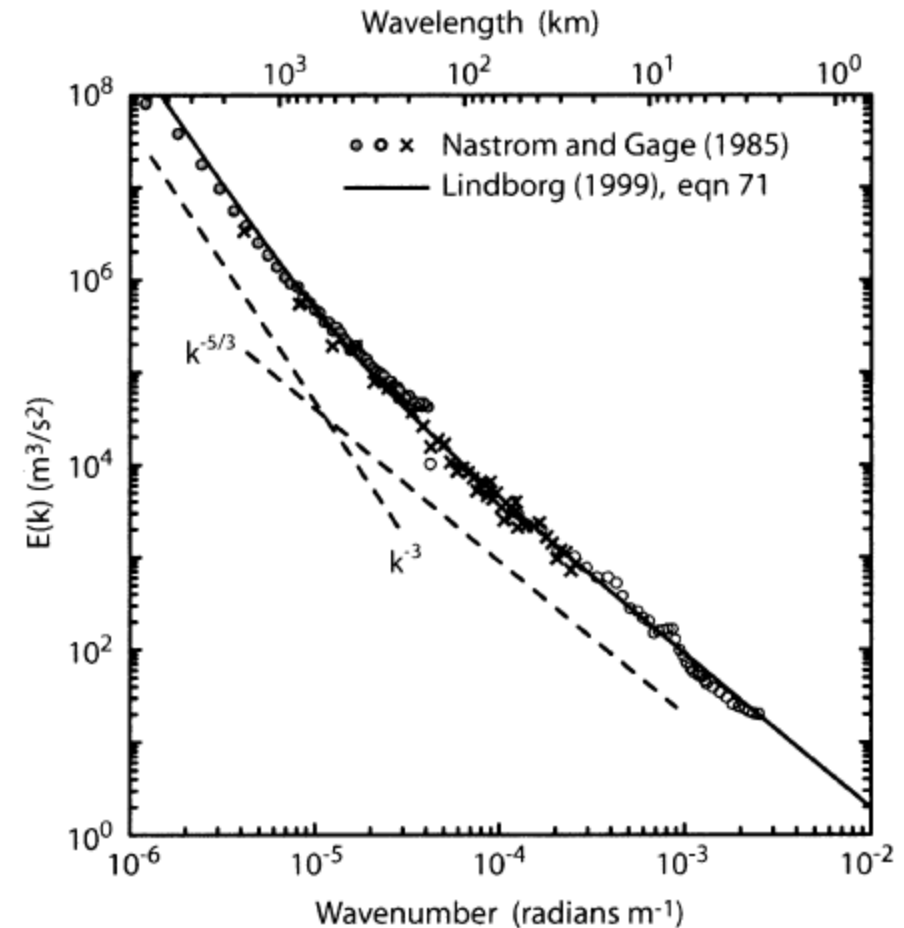


FIG. 1. Nastrom and Gage (1985) spectrum derived from the GASP aircraft observations (symbols) and the Lindborg (1999) functional fit to the MOZAIC aircraft observations.

- In order to fulfil the observed energy cascade from resolved scales to unresolved scales in GCMs, a **dissipation term** is added:

$$Dissip(\psi) = \frac{(-1)^{q+1}}{\tau} \nabla^{2q} \psi$$

Lateral dissipation in GCMs as a tool to pin the energy cascade

From *Numerical Techniques for Global Atmospheric Models*, Lauritzen et al. (eds), 2010

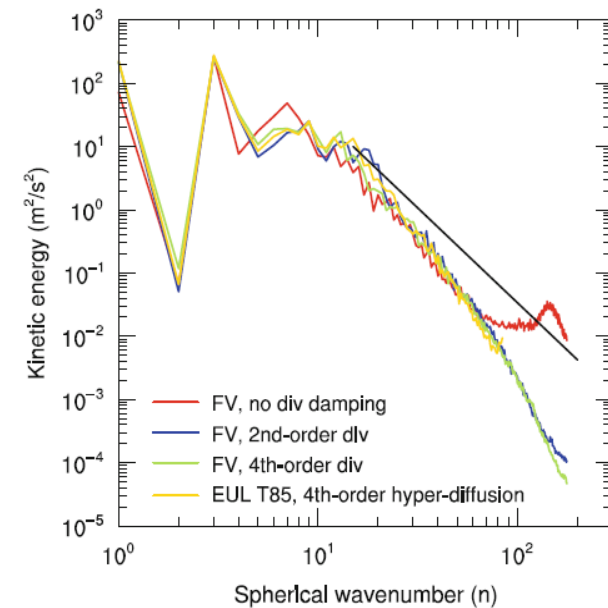
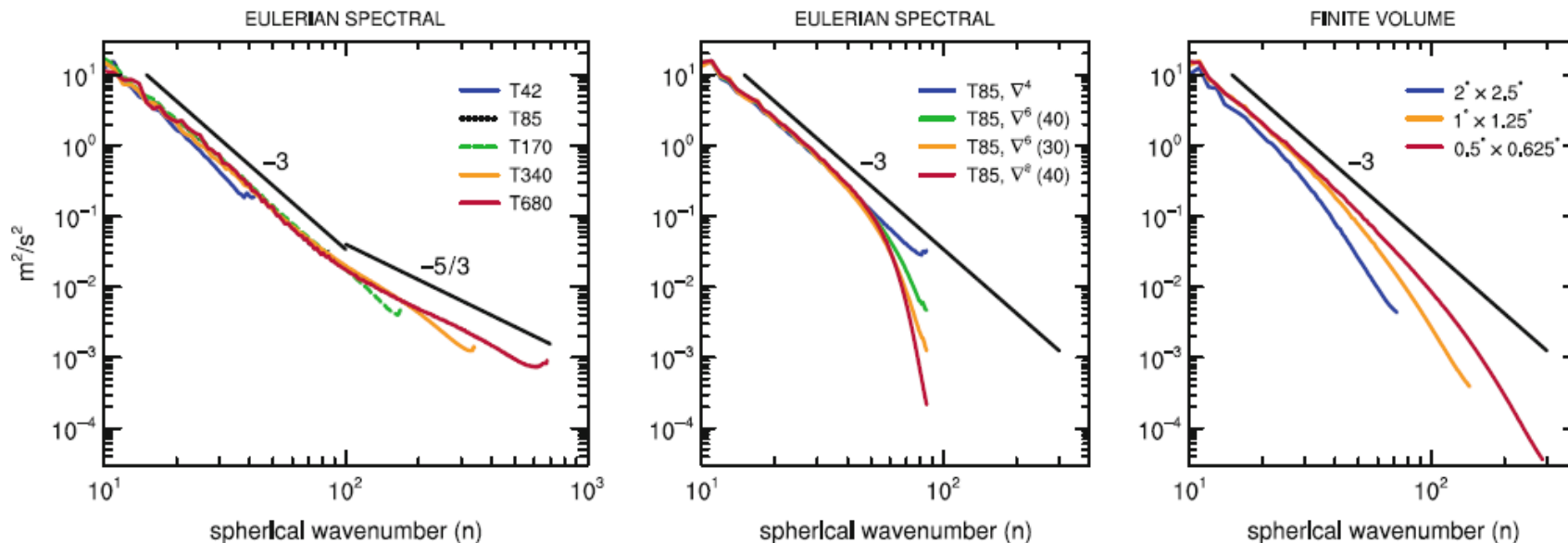


Fig. 13.4 250 hPa kinetic energy spectra as a function of the spherical wavenumber (n) in aqua-planet simulations from (left) CAM Eulerian spectral dynamical core with ∇^4 diffusion for different resolutions, (center) T85L26 Eulerian spectral dynamical with ∇^4 , ∇^6 and ∇^8 diffusion, and (right) CAM Finite Volume (FV) dynamical core for different $lat \times lon$ resolutions in degrees and 26 levels

Illustrative example of dissipation

- Simple 1D diffusion equation toy model:

$$\frac{dA}{dt} = \nu \frac{\partial^2 A}{\partial x^2}$$

- Von Neumann (Fourier mode) analysis

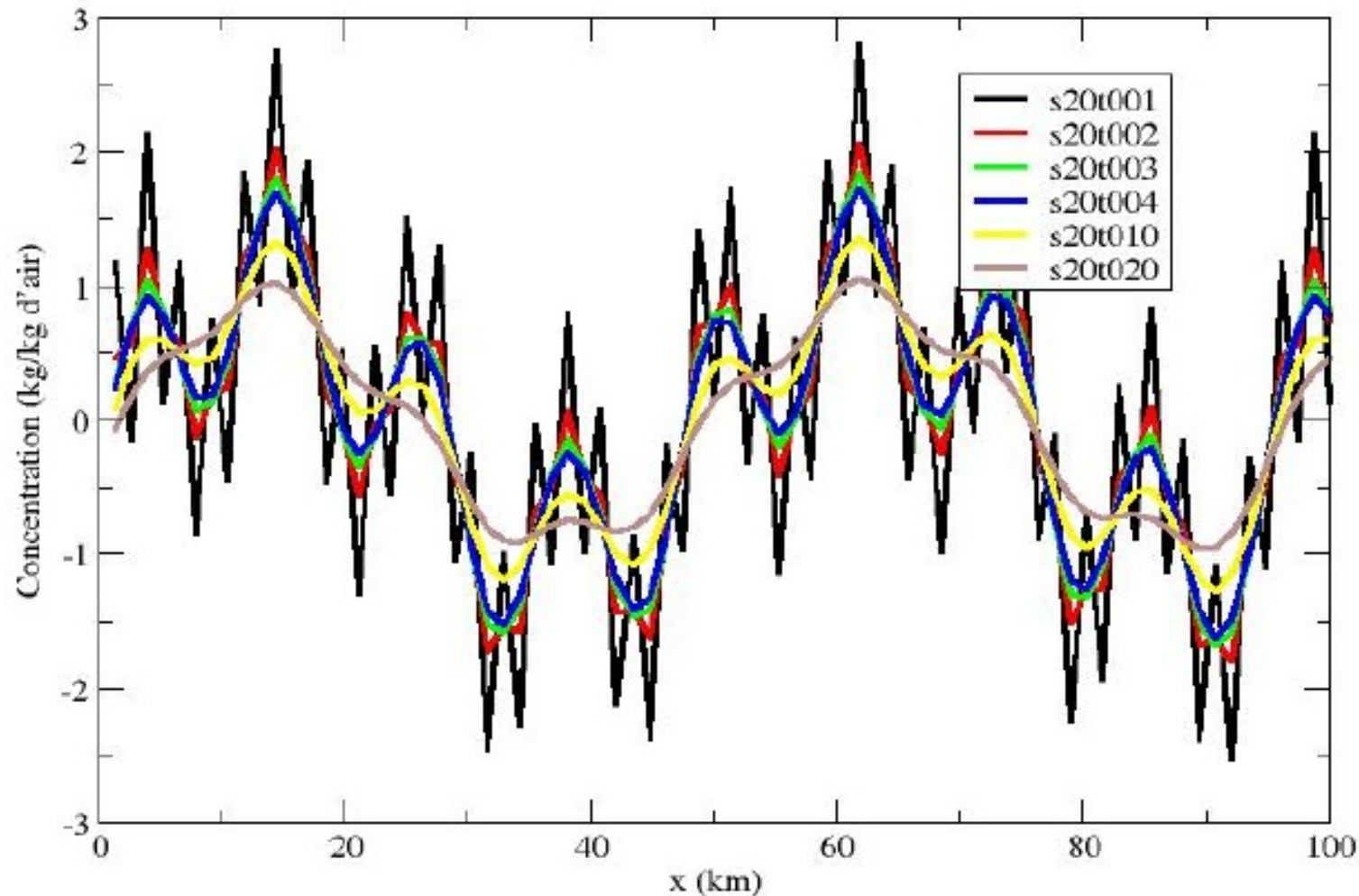
$$A_k(t) = a_k(t) \cdot \sin(kx)$$

- Explicit Euler time marching (with stability condition!):

$$a_k^{n+1} = (1 - \nu k^2 \Delta t) a_k^n$$

Note that mode damping is stronger for large k

Illustrative example of dissipation



- Temporal evolution, from an initial condition consisting of 2 sine modes and an extreme (2 grid points wavelength) “numerical mode”

Controlling dissipation in LMDZ

- Parameters in file gcm.def:

dissip_period: Apply dissipation every dissip_period dynamical steps
(or **specify 0 to let model pick an appropriate value**)

nitergdiv: number of iterations on velocity dissipation operator grad.div

nitergrot: number of iterations on velocity dissipation operator grad.rot

niterh: number of iterations on temperature dissipation operator
div.grad

Usual values: nitergdiv=1, nitergrot=2, niterh=2

tetagdiv: dissipation time scale (s) for smallest wavelength for u,v
(grad.div component)

tetagrot: dissipation time scale (s) for smallest wavelength for u,v
(grad.rot component)

tetatemp: dissipation time scale (s) for smallest wavelength for
potential temperature (div.grad)

values depend on horizontal resolution

Controlling dissipation in LMDZ

- Parameters in file gcm.def:

tetagdiv: dissipation time scale (s) for smallest wavelength for u,v (grad.div component)

tetagrot: dissipation time scale (s) for smallest wavelength for u,v (grad.rot component)

tetatemp: dissipation time scale (s) for smallest wavelength for potential temperature (div.grad)

optimal tetat values depend on horizontal resolution

- Moreover there is a multiplicative factor for the dissipation coefficient, which increases with model levels (see dyn3d_common/**inidissip.F90**), which can be controlled by flag “vert_prof_dissip”

The sponge layer

- In addition to lateral dissipation, it is necessary to damp vertically propagating waves (non-physically reflected downward from model top).
- The **sponge layer is limited to topmost layers** (usually 4) and added during the dissipation step.
- Sponge modes and parameters (gcm.def):
 - iflag_top_bound**: 0 for no sponge, 1 for sponge over 4 topmost layers, 2 for sponge from top to 100 times topmost layer pressure
 - mode_top_bound**: 0 for no relaxation, 1 to relax u,v to zero, 2 to relax u,v to their zonal mean, 3 to relax u,v and potential temperature to their zonal mean.
 - tau_top_bound**: inverse of characteristic time scale at the topmost layer (halved at each successive descending layer)

Some examples of typical values for various resolutions

From test simulations using various horizontal resolutions (Foujols et al.)

http://forge.ipsl.jussieu.fr/igcmg/wiki/ResolutionIPSLCM4_v2

Tableau récapitulatif des différences entre résolutions et avec le couplé standard

	R44	R97	R99	R149	R1414	R1914
Résolutions						
atmosphère	44x43x19	96x71x19	96x95x19 (*)	144x96x19	144x142x19	192x142x19 (*)
océan	ORCA2	ORCA2	ORCA2	ORCA2	ORCA2	ORCA2
Etat initial						
océan	2L20 - 20 ans 30 12 1859	2L20 -30 12 1859	2L20 -30 12 1859	2L20 - 30 12 1859	2L20 - 30 12 1859	2L20 - 30 12 1859
atmosphère	10 ans forcé AMIP 1980-1989 F44A	10 ans forcé AMIP 1980-1989 F97A	10 ans forcé AMIP 1980-1989 F99A	10 ans forcé AMIP 1980-1989 F149A	10 ans forcé AMIP 1980-1989 F1414A	10 ans forcé AMIP 1980-1989 F1914A
coupleur	défaut	défaut	défaut	défaut	défaut	défaut
atmosphère (idem avec dynzon)	10 ans forcé AMIP 1980-1989 F44AZ	10 ans forcé AMIP 1980-1989 F97AZ	10 ans forcé AMIP 1980-1989 F99AZ	10 ans forcé AMIP 1980-1989 F149AZ	10 ans forcé AMIP 1980-1989 F1414AZ	10 ans forcé AMIP 1980-1989 F1914AZ
coupleur	défaut	défaut	défaut	défaut	défaut	défaut
Paramètres						
day_step	240	480	480 (=96x71)	720	720	960 (**)
nitergdiv	1					
iphysiq	5	10	10 (=)	15	15	20 (**)
tetagdiv	36000.	5400. (**)	5400. (=)	5400. (**)	3600.	3600. (=144x142)
tetagrot	18000.	7200. (**)	7200. (=)	7200. (**)	5400.	5400. (=)
tetatemp	18000.	7200. (**)	7200. (=)	7200. (**)	5400.	5400. (=)
cvl_corr	1.0	1.0	1.0 (=)	1.0	1.0 (**)	1.0 (=)

Rules of thumb for run.def parameters

- Time steps in LMDZ:

dynamical time steps: $\text{dtvr} = \text{daysec} / \text{day_step}$

physics time step: $\text{dtphys} = \text{iphysiq} * \text{dtvr}$

dissipation time step: $\text{dtdiss} = \text{dissip_period} * \text{dtvr}$

tracer advection time step: $\text{dtvrtrac} = \text{iapp_trac} * \text{dtvr}$

- Constraints to be aware of:

dtvr limited by CFL for waves: $\text{Cmax}.\text{dt} < \min(\text{dx}, \text{dy})$

dtrtrac limited by advection CFL: $\text{Umax}.\text{dt} < \min(\text{dx}, \text{dy})$

iphysiq , dtvrtrac , dissip_period should be multiples of ipperiod

Rules of thumb for run.def parameters

- Constraints to be aware of (continued):
dissipation time step should be much smaller than dissipation timescales:
$$\text{dtdiss} \ll \text{tetatgdiv}, \text{tetagrot}, \text{tetatemp}$$
- Changing time step with resolution on a regular grid:
$$\text{day_step}(\max(\text{iim}, \text{jjm})=N) \sim \text{day_step}(\max(\text{iim}, \text{jjm})=M) * M/N$$
- Time step for a zoomed simulation, compared to regular grid:
$$\text{day_step}(\text{zoom}) \sim \text{day_step}(\text{regular}) * \max(\text{grossismx}, \text{grossismy})$$