

# Atmosphere-surface coupling in LMDZ

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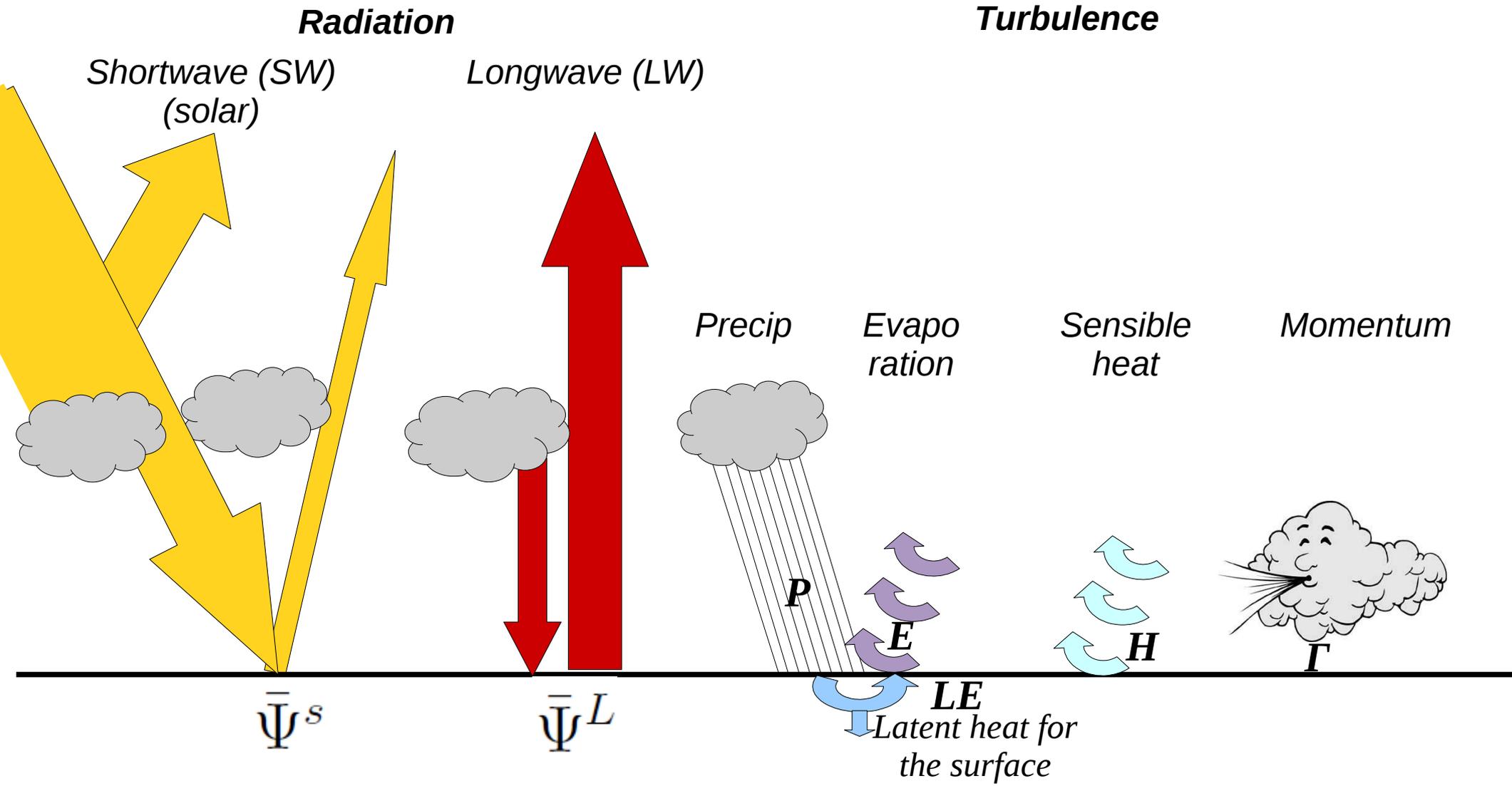
**Technical note** : Description of the interface with the surface and  
the computation of the turbulent diffusion in LMDZ  
[http://www.lmd.jussieu.fr/~jldufres/publi/pbl\\_surface.pdf](http://www.lmd.jussieu.fr/~jldufres/publi/pbl_surface.pdf)

*LMDZ training, 6 December 2016*

# Atmosphere-surface interactions

The **atmosphere and the surface are coupled** through *turbulence* (in boundary layer) and *radiation* (SW and LW). Currently, there are no direct influence of the surface to other parametrizations.

The surface “receive” precipitation from the atmosphere (no direct feedback).



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The **atmosphere and the surface are coupled** through *turbulence* (in boundary layer) and *radiation* (SW and LW). Currently, there are no direct influence of the surface to other parametrizations.

The surface “receive” precipitation from the atmosphere (no direct feedback).

The surface impacts the atmosphere via the orography (factors constant with time)

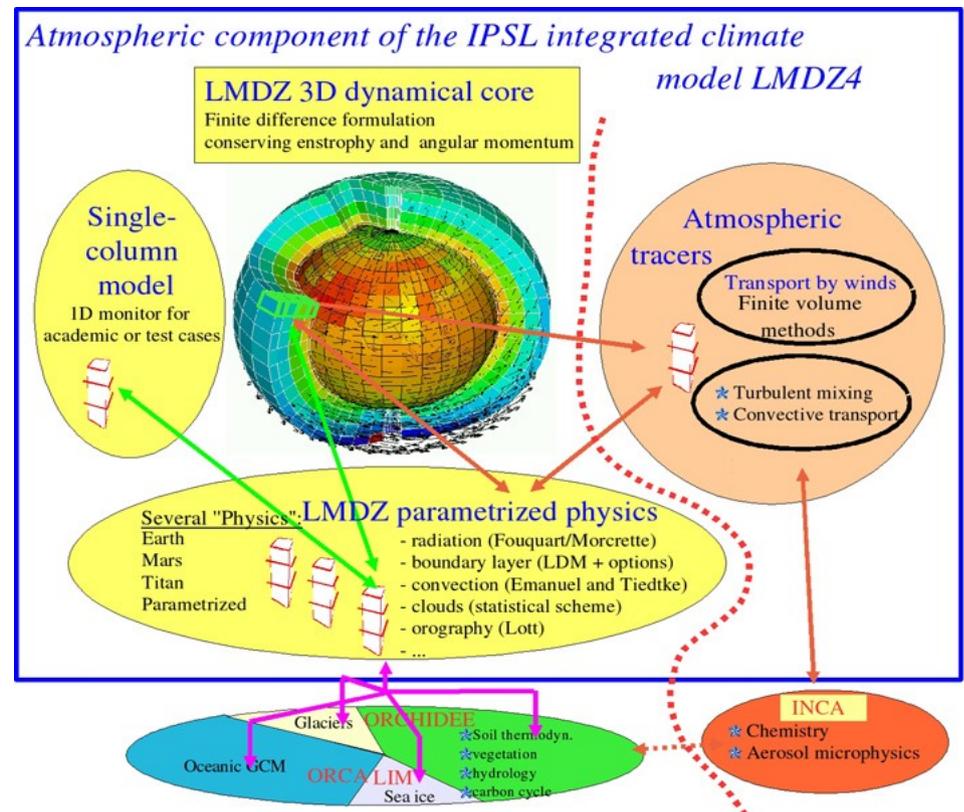
## In LMDZ:

Each surface grid may be decomposed in a maximum of 4 sub-grids of different types: land (\_ter), continental ice (\_lic), open ocean (\_oce), sea-ice (\_sic)

*Radiation* depends only on mean surface properties

*Turbulent diffusion* depends on local sub-grid property

No influence of sub-surface properties to any other parameterization.



# Turbulent diffusion

The change of a variable  $X$  with time is:

$$\rho \partial_t X = -\partial_z \phi \quad (8)$$

where  $\rho$  is the volumic mass ( $kg.m^{-3}$ ) and  $\phi$  the flux of  $X$ . Variable  $X$  can be the specific humidity, moist static energy, momentum, tracer.... For the vertical diffusion, the flux  $\phi$  of  $X$  is defined as:

$$\phi = -\rho k_z \partial_z X \quad (9)$$

with  $k_z$  is the diffusion coefficient ( $m^2.s^{-1}$ ).

**Space discretization.** We consider  $n$  layers from  $l = 1$  (surface) to  $l = n$  (top of atmosphere, TOA) and  $n + 1$  interfaces from  $l = 1$  (surface) to  $l = n + 1$  (TOA). The space discretization of above equations gives

$$m_l \partial_t X_l = \phi_l - \phi_{l+1} \quad (10)$$

$$\phi_l = -K_l (X_l - X_{l-1}) \quad (11)$$

with  $X_l$  average value of  $X$  for layer  $l$ ,  $m_l$  mass per unit surface ( $kg.m^{-2}$ ) of layer  $l$ ,  $\phi_l$  flux at interface  $l$  and

$$K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l} \quad (12)$$

with  $P_l$  pressure of layer  $l$ .

# Turbulent diffus

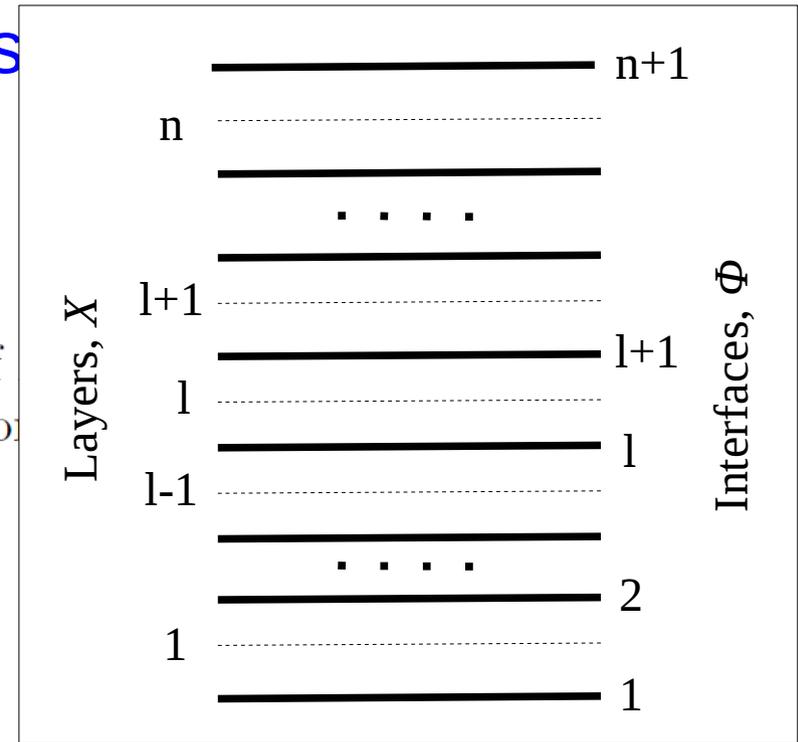
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## Turbulent diffusion

$$\begin{aligned} m_l \partial_t X_l &= \phi_l - \phi_{l+1} \\ \phi_l &= -K_l (X_l - X_{l-1}) \end{aligned}$$

**Time discretization.** We use an implicit scheme:

$$m_l \frac{X_l(t + \delta t) - X_l(t)}{\delta t} = \phi_l(t + \delta t) - \phi_{l+1}(t + \delta t)$$

$$\begin{aligned} m_l \frac{X_l - X_l^0}{\delta t} &= \phi_l - \phi_{l+1} \\ &= K_{l+1} (X_{l+1} - X_l) - K_l (X_l - X_{l-1}) \end{aligned}$$

with :  $X_l = X_l(t + \delta t)$  and  $X_l^0 = X_l(t)$ .

$$\left( \frac{m_l}{\delta t} + K_{l+1} + K_l \right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

Tridiagonal system that can be solved for vector  $X$

$$-K_l X_{l-1} + \left( \frac{m_l}{\delta t} + K_{l+1} + K_l \right) X_l + K_{l+1} X_{l+1} = \frac{m_l}{\delta t} X_l^0$$

## Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$\left(\delta P_l + R_{l+1}^X + R_l^X\right) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1} \quad (2 \leq l < n)$$

$$\text{with } R_l^X = g\delta t K_l$$

At the top

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

At the bottom:

$$m_1 \frac{X_1 - X_1^0}{\delta t} = K_2(X_2 - X_1) - F_1^X$$

$$\left(\delta P_1 + R_1^X\right) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t F_1^X$$

With  $F_1^X$  : flux of  $X$  at the bottom of the first layer (i.e. between the surface and the atmosphere), positive downward.

## Solving the tridiagonal system

Starting from top:

$$(\delta P_n + R_n^X) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

can be written as:

$$X_n = C_n^X + D_n^X X_{n-1}$$

with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$

$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$

## Solving the tridiagonal system

$$X_{l+1} = C_{l+1}^X + D_{l+1}^X X_l$$

$$(\delta P_l + R_{l+1}^X + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1} \quad (2 \leq l < n)$$

$$(\delta P_l + R_{l+1}^X (1 - D_{l+1}^X) + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X C_{l+1}^X + R_l^X X_{l-1}$$

So we obtain by recurrence:

$$X_l = C_l^X + D_l^X X_{l-1} \quad (2 \leq l \leq n)$$

with, for  $(2 \leq l < n)$

$$C_l^X = \frac{X_l^0 \delta P_l + R_{l+1}^X C_{l+1}^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

$$D_l^X = \frac{R_l^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

## Solving the tridiagonal system

**At the bottom of the boundary layer**  $X_2 = C_2^X + D_2^X X_1$

$$(\delta P_1 + R_2^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t F_1^X$$

replacing  $X_2$  in the equation above:

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

with

$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X (1 - D_2^X)}$$

$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$

If  $F_1$  is known, then  $X_1$  and all other  $X_i$  are known, and also the flux  $\Phi_i$

# Coupling with the surface

Equation for the variables of the first layer

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

Sensitivity of  $X_1$  to the flux at surface  $F_1$   
Value if the flux  $F_1 = 0$

**Atmospheric model:** Boundary layer:

$$X_1 = f(F_1^X)$$

**Surface model:**

Flux between atmosphere and surface:

$$F_1^X = g(X_1, X_s) \Rightarrow F_1^X = g(f(F_1^X), X_s) \Rightarrow F_1^X = \hat{g}(X_s)$$

Surface variables are prescribed or computed (budget at surface):

$$X_s = h(F_1^X) \Rightarrow X_s = h(\hat{g}(X_s)) \Rightarrow X_s = \dots$$

# Coupling with the surface

Equation for the variables of the first layer

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

Sensitivity of  $X_1$  to the flux at surface  $F_1$   
Value if the flux  $F_1=0$

$X$ : temperature  $T$ , humidity  $q$ , velocity  $V_x$  and  $V_y$

$F^x$ : Flux of heat, flux of water mass or flux of momentum

Typical expression of the flux with the surface:  $F_1 = K_1 (X_1 - X_s)$

**Each surface model has to compute  $X_s$  and  $F_1$  using  $X_1$ ,  $A_1$  and  $B_1$**

If  $F_1$  is known, then  $X_1$  and all other  $X_l$  are known, and also the flux  $\Phi_l$

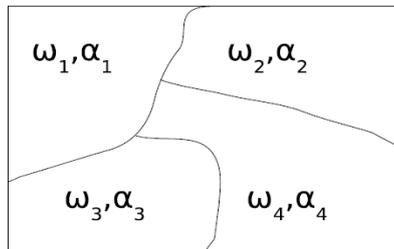
In LMDZ, the turbulent flux are computed separately over each sub-surface type

# Coupling between atmospheric column(s) and sub-surfaces

Each grid cell is divided into several sub-areas or "sub-surfaces" of fractions  $\omega_i$

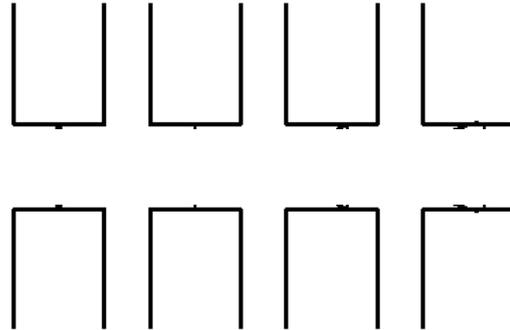
Sub-surfaces

$$\sum_i \omega_i = 1$$



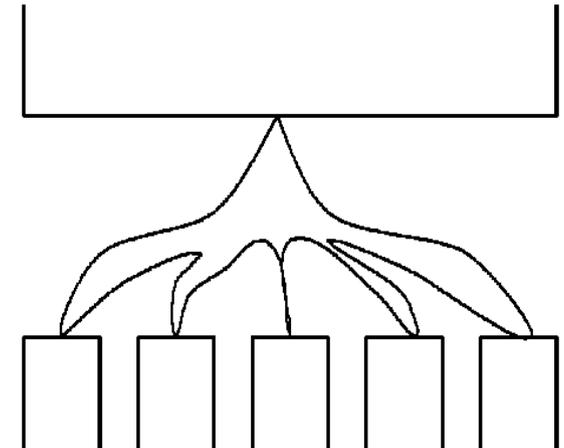
Turbulent flux

One PBL over **each** sub-surface



Radiative flux

One column **covers all** the sub-surface



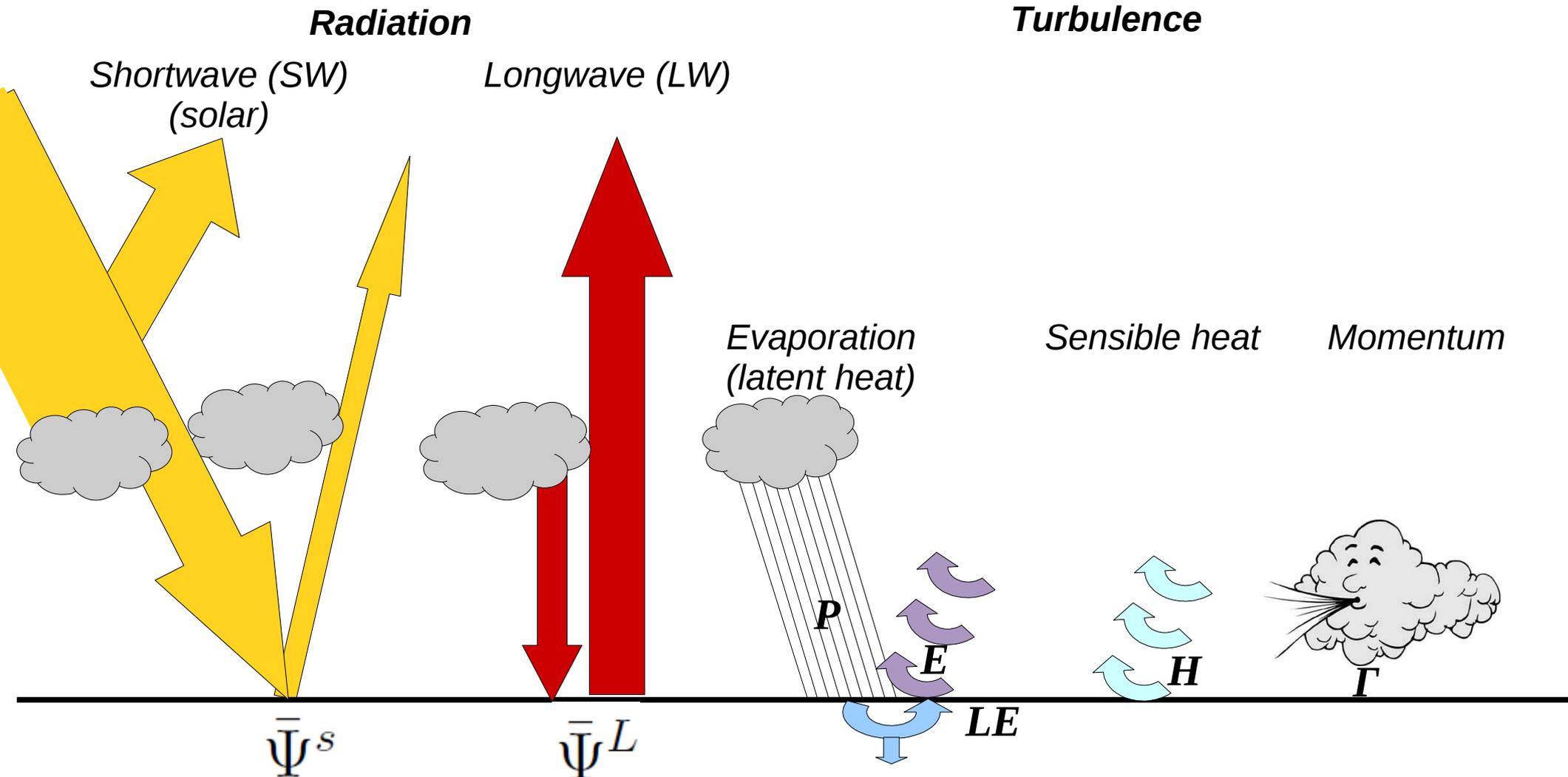
**Each sub surface has to compute  $F_1$  using variables  $X_1$ ,  $A_1$  and  $B_1$**

The boundary layer tendencies in the atmosphere are mixed between subcolumns (equivalent of averaging the surface flux)

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# Derivation of local sub-surface **net solar radiation** from grid average net solar radiation

The grid average net flux  $\Psi_s$  at surface has been computed for each grid point by the radiative code.

We want (1) to conserve energy and (2) to take into account the value of the local albedo  $\alpha_i$  of the sub-surface.

We compute the downward SW radiation as  $F_{\downarrow}^s = \frac{\Psi_s}{(1 - \alpha)}$

with the mean albedo  $\alpha = \sum_i \omega_i \alpha_i$

**For each sub-surface i**, the absorbed solar radiation reads:  $\psi_i^s = (1 - \alpha_i) F_{\downarrow}^s$

$$\psi_i^s = \frac{(1 - \alpha_i)}{(1 - \alpha)} \bar{\Psi}^s$$

One may verified that this procedure ensure energy conservation, i.e.  $\sum_i \omega_i \psi_i^s = \Psi_s$

# Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

The net longwave (LW) radiation  $\bar{\Psi}^L$  has been computed by the radiative code for each grid cell. How to split it depending on the sub-surfaces local properties and ensuring energy conservation?

If the downward longwave flux  $F_{\downarrow}$  is uniform within each grid, the net LW flux for a sub-surface  $i$  may be written as:

$$\psi_i^L(T_i) = \epsilon_i (F_{\downarrow} - \sigma T_i^4) \quad (1)$$

where  $T_i$  is the surface temperature of sub-surface  $i$  and  $\epsilon_i$  its emissivity. A linearization around the mean temperature  $\bar{T}$  gives:

$$\psi_i^L(T_i) \approx \epsilon_i (F_{\downarrow} - \sigma \bar{T}^4) - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T}) \quad (2)$$

To conserve the energy, the following relationship must be true:

$$\sum_i \omega_i \psi_i^L = \bar{\Psi}^L \quad (3)$$

Using Eq. 2 gives

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i (T_i - \bar{T}) \quad (4)$$

where  $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$  is the mean emissivity.

# Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i (T_i - \bar{T}) \quad (4)$$

where  $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$  is the mean emissivity. The second term on the right hand side is zero if

$$\bar{T} = \frac{\sum_i \omega_i \epsilon_i T_i}{\bar{\epsilon}} \quad (5)$$

To ensure energy conservation, we need in addition to verify:

$$\bar{\Psi}^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) \quad (6)$$

Which is consistent with the definition of the net LW flux at the surface. We rewrite now Eq. 2 as:

$$\psi_i^L(T_i) \approx \frac{\epsilon_i}{\bar{\epsilon}} \bar{\Psi}^L - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T}) \quad (7)$$

Due to radiative code limitation, in LMDZ, we always must have  $\epsilon_i = 1$

# Call tree

## In subroutine PHYSIQ

loop over time steps

CALL change\_srf\_frac : Update fraction of the sub-surfaces (pctsrfr)

....

**CALL pbl\_surface** Main subroutine for the interface with surface

Calculate net radiation at sub-surface

*Loop over the sub-surfaces nsrf*

Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface is not zero)

CALL clcdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)

CALL coef\_diff\_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)

CALL climb\_hq\_down downhill for henthalpie H and humidity Q

CALL climb\_wind\_down downhill for wind (U and V)

CALL surface models for the various surface types: **surf\_land, surf\_landice, surf\_ocean or surf\_seaice**. **Each surface model computes:**

- evaporation, latent heat flux, sensible heat flux
- surface temperature, albedo

CALL climb\_hq\_up : compute new values of henthalpie H and humidity Q

CALL climb\_wind\_up : compute new values of wind (U and V)

Uncompress variables : (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)

Cumulate in global variables after weighting by sub-surface fractions

Surface diagnostics : (T2m, Q2m, wind at 10m...)

*End Loop over the sub-surfaces*

Calculate the mean values over all sub-surfaces for some variables

**End pbl-surface**