# Interface atmosphère-surface et résolution de la diffusion turbulente dans LMDZ

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Il existe une note interne sur le sujet

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### Atmosphere-surface interactions

The **atmosphere and the surface are coupled** through *turbulent diffusion* (in boundary layer) and *radiation* (SW and LW). Currently, there are no direct influence of the surface to other parametrizations.

The surface impacts the atmosphere via the orography (factors constant with time)

The surface "receive" precipitation from the atmosphere (no direct feedback).

#### In LMDZ:

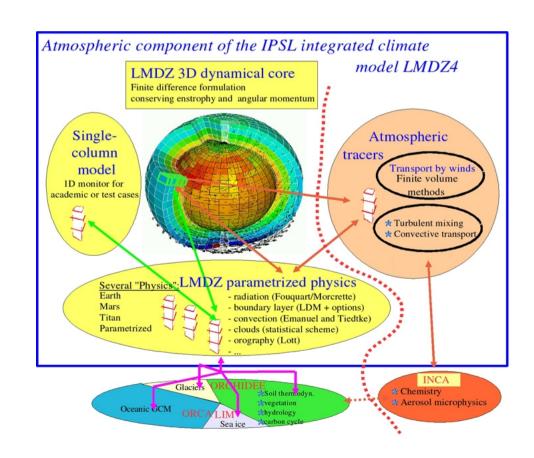
Each surface grid may be decomposed in a maximum of 4 sub-grids of different types:

land (\_ter), continental ice (\_lic), open ocean (\_oce), sea-ice (\_sic)

Radiation depends only on mean surface properties

*Turbulent diffusion* depends on local subgrid property

No influence of sub-surface properties to any other parameterization.



## Derivation of local sub-surface **net solar radiation** from grid average net solar radiation

The grid average net flux  $\Psi_s$  at surface has been computed for each grid point by the radiative code.

We want (1) to conserve energy and (2) to take into account the value of the local albedo  $\alpha_{i}$  of the sub-surface.

We compute the downward SW radiation as  $F^s_{\downarrow} = \frac{\Psi_s}{(1-\alpha)}$  with the mean albedo  $\alpha = \sum_i \omega_i \alpha_i$ 

For each sub-surface, the absorbed solar radiation reads:  $\psi_i^s = (1 - \alpha_i) F_{\downarrow}^s$ 

$$\psi_i^s = \frac{(1 - \alpha_i)}{(1 - \alpha)} \bar{\Psi}^s$$

One may verified that this procedure ensure energy conservation, i.e.  $\sum_i \omega_i \psi_i^s = \Psi_s$ 

## Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

The net longwave (LW) radiation  $\bar{\Psi}^L$  has been computed by the radiative code for each grid cell. How to split it depending on the sub-surfaces local properties and ensuring energy conservation?

If the downward longwave flux  $F_{\downarrow}$  is uniform within each grid, the net LW flux for a sub-surface i may be written as:

$$\psi_i^L(T_i) = \epsilon_i \left( F_{\downarrow} - \sigma T_i^4 \right) \tag{1}$$

where  $T_i$  is the surface temperature of sub-surface i and  $\epsilon_i$  its emissivity. A linearization around the mean temperature  $\bar{T}$  gives:

$$\psi_i^L(T_i) \approx \epsilon_i \left( F_{\downarrow} - \sigma \bar{T}^4 \right) - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T})$$
 (2)

To conserve the energy, the following relationship must be true:

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\Psi}^{L} \tag{3}$$

Using Eq. 2 gives

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\epsilon} \left( F_{\downarrow} - \sigma \bar{T}^{4} \right) - 4\sigma \bar{T}^{3} \sum_{i} \omega_{i} \epsilon_{i} \left( T_{i} - \bar{T} \right) \tag{4}$$

where  $\bar{\epsilon} = \sum_{i} \omega_{i} \epsilon_{i}$  is the mean emissity.

## Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\epsilon} \left( F_{\downarrow} - \sigma \bar{T}^{4} \right) - 4\sigma \bar{T}^{3} \sum_{i} \omega_{i} \epsilon_{i} \left( T_{i} - \bar{T} \right)$$

$$\tag{4}$$

where  $\bar{\epsilon} = \sum_{i} \omega_{i} \epsilon_{i}$  is the mean emissity. The second term on the right hand side is zero if

$$\bar{T} = \frac{\sum_{i} \omega_{i} \epsilon_{i} T_{i}}{\bar{\epsilon}} \tag{5}$$

To ensure energy conservation, we need in addition to verify:

$$\bar{\Psi}^L = \bar{\epsilon} \left( F_{\downarrow} - \sigma \bar{T}^4 \right) \tag{6}$$

Which is consistent with the definition of the net LW flux at the surface. We rewrite now Eq. 2 as:

$$\psi_i^L(T_i) \approx \frac{\epsilon_i}{\bar{\epsilon}} \bar{\Psi}^L - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T})$$
 (7)

Due to radiative code limitation, in LMDZ, we always must have  $\varepsilon_i = 1$ 

#### **Turbulent diffusion**

The change of a variable X with time due to vertical diffusion is:

$$\rho \partial_t X = -\partial_z \phi \tag{8}$$

where  $\rho$  is the volumic mass  $(kg.m^{-3})$ . Variable X can be the specific humidity, moist static energy, momentum, tracer....  $\phi$  is the flux of X and is defined as:

$$\phi = -\rho k_z \partial_z X \tag{9}$$

with  $k_z$  is the diffusion coefficient  $(m^2.s^{-1})$ .

**Space discretization.** We consider n layers from l = 1 (surface) to l = n (top of atmosphere, TOA) and n+1 interfaces from l=1 (surface) to l=n+1 (TOA). The space discretization of above equations gives

$$m_l \partial_t X_l = \phi_l - \phi_{l+1}$$

$$\phi_l = K_l (X_{l-1} - X_l)$$

$$(10)$$

$$(11)$$

$$\phi_l = K_l(X_{l-1} - X_l) \tag{11}$$

with  $X_l$  average value of X for layer l,  $m_l$  mass per unit surface  $(kg.m^{-2})$  of layer l,  $\phi_l$  flux at interface l and

$$K_l = -\frac{k_z \rho^2 g}{P_l - P_{l-1}} \tag{12}$$

with  $P_l$  pressure at interface l.

### Turbulent diffus

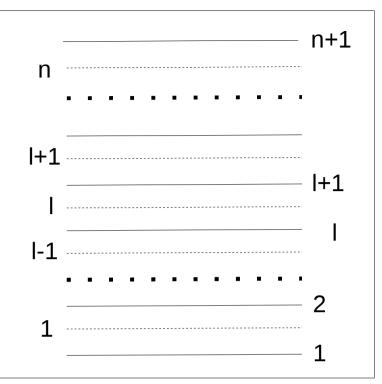
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#### **Turbulent diffusion**

$$m_l \partial_t X_l = \phi_l - \phi_{l+1}$$
$$\phi_l = K_l (X_{l-1} - X_l)$$

**Time discretization.** We use an implicit scheme:

$$m_l \frac{X_l(t+\delta t) - X_l(t)}{\delta t} = \phi_l(t+\delta t) - \phi_{l+1}(t+\delta t)$$
(13)

$$m_{l} \frac{X_{l} - X_{l}^{0}}{\delta t} = \phi_{l} - \phi_{l+1}$$

$$= K_{l+1}(X_{l+1} - X_{l}) - K_{l}(X_{l} - X_{l-1})$$
(14)

$$= K_{l+1}(X_{l+1} - X_l) - K_l(X_l - X_{l-1})$$
(15)

with the following notations:  $X_l = X_l(t + \delta t)$  and  $X_l^0 = X_l(t)$ . This equations may be written as:

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$
(16)

Tridiagonal system that can be solved for vector X

## Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$\left(\delta P_l + R_{l+1}^X + R_l^X\right) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1} \quad (2 \le l < n)$$

with 
$$R_l^X = g\delta t K_l$$

At the top

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

At the bottom

$$(\delta P_1 + R_1^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t F_1^X$$

## Solving the tridiagonal system

Starting from top, one can obtained by reccurence:

$$X_{l} = C_{l}^{X} + D_{l}^{X} X_{l-1} \qquad (2 \le l \le n)$$

with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$
$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$

for 
$$(2 \le l < n)$$

$$C_l^X = \frac{X_l^0 \delta P_l + R_{l+1}^X C_{l+1}^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$
$$D_l^X = \frac{R_l^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

## Solving the tridiagonal system

At the bottom

$$X_2 = C_2^X + D_2^X X_1$$

$$(\delta P_1 + R_1^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t F_1^X$$

using Eq. 33 to replace  $X_2$  in the equation above:

$$X_1 = A_1^X + B_1^X . F_1^X . \delta t$$

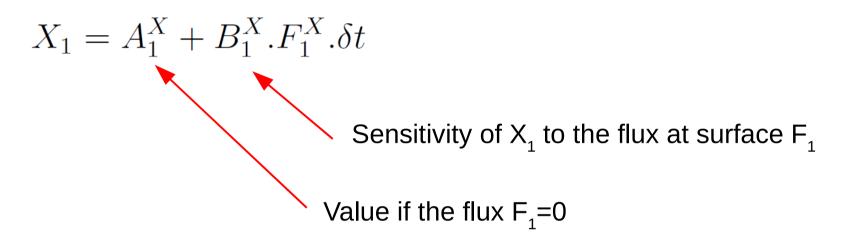
with

$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X (1 - D_2^X)}$$
$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$

If  $F_1$  is known, then  $X_1$  and all other  $X_1$  are known, and also the flux  $\Phi_1$ 

### Coupling with the surface

Equation for the variables of the first layer



Typical expression of the flux with the surface  $F_1 = K_1 (T_1-T_s)$ 

Each surface model has to compute the flux  $F_1$  using variables  $X_1$ ,  $A_1$  and  $B_1$ 

In LMDZ, the turbulent flux are computed separately over each sub-surface type

#### In subroutine PHYSIQ

### Call tree

loop over time steps

CALL change\_srf\_frac : Update fraction of the sub-surfaces (pctsrf)

...

**CALL pbl\_surface** Main subroutine for the interface with surface

Calculate net radiation at sub-surface

Loop over the sub-surfaces nsrf

Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface in not zero)

CALL clcdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)

CALL coef\_diff\_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)

CALL climb\_hq\_down downhill for henthalpie H and humidity Q

CALL climb\_wind\_down downhill for wing (U and V)

CALL surface models for the various surface types: surf\_land, surf\_ocean or surf\_seaice. Each surface model computes:

- evaporation, latent heat flux, sensible heat flux
- surface temperature, albedo

CALL climb hq up: compute new values of henthalpie H and humidity Q

CALL climb\_wind\_up : compute new values of wind (U and V)

Uncompress variables: (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)

Cumulate in global variables after weighting by sub-surface fractions Surface diagnostics: (T2m, Q2m, wind at 10m...)

End Loop over the sub-surfaces

Calculate the mean values over all sub-surfaces for some variables

**End pbl-surface**