

Atmosphere-surface interactions in IPSL-CM

In LMDZ:

Each surface grid can be decomposed in a maximum of 4 sub-grid of different type: land (_ter), continental ice (_lic), open ocean (_oce) and sea_ice (_sic)

Radiation at the surface depends on mean surface properties (albedo, emissivity)

Turbulent diffusion depends on local sub-grid properties but each sub-surface sees the same atmosphere

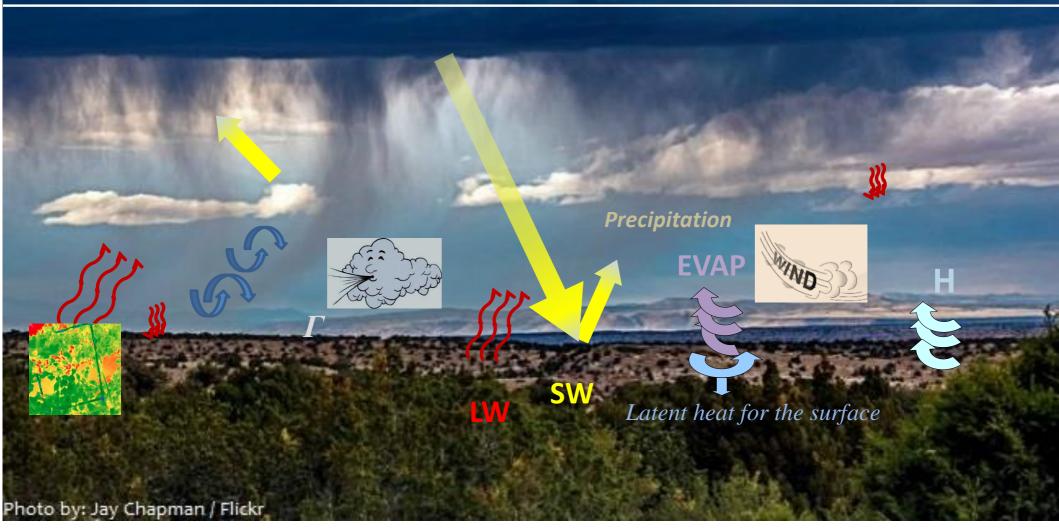












The **atmosphere and the surface are coupled** through *turbulence* (in boundary layer) and *radiation* (SW and LW).

Surface impacts atmosphere via orography, roughness, albedo, emissivity

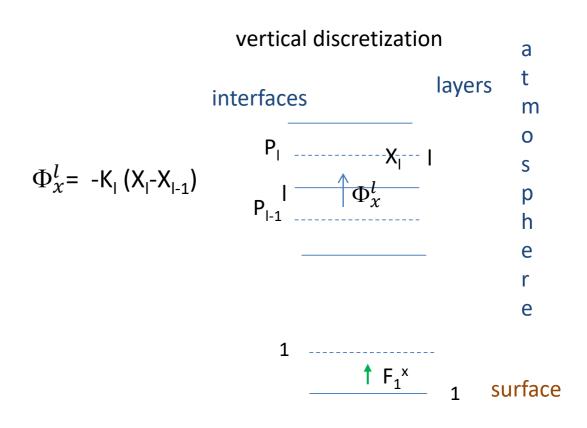
Change of a variable X with the time due to the turbulent transport

(continuity):
$$\frac{\partial X}{\partial t} = -\frac{1}{\rho} \frac{\partial \Phi}{\partial z}$$

$$\Phi = -\rho k_z \frac{\partial X}{\partial z}$$

Turbulent diffusion (pbl_surface, LMDZ)

$$\Phi = -\rho k_z \frac{\partial X}{\partial z}$$



X= specific humidity, enthalpie, momentum

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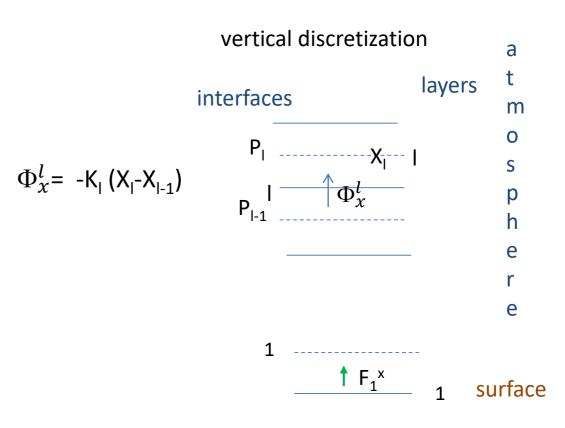
$$F_{1}^{q} = \beta \rho VC_{d,q} (q_{1} - q_{s}(T_{s}))$$

$$F_{1}^{h} = \rho VC_{d,h} (T_{1} - T_{s})$$

$$F_{1}^{u} = \rho VC_{d,m} (u_{1} - u_{o})$$

$$C_{dm} = \kappa^2 / (\ln(\frac{z}{z_{0m}}) * \ln(\frac{z}{z_{0m}})) * F_{stab}$$

$$C_{dh} = \kappa^2 / (\ln(\frac{z}{z_{0m}}) * \ln(\frac{z}{z_{0h}})) * F_{stab}$$



X= specific humidity, enthalpie, momentum

$$\delta P = (P_{l-1} - P_l) = \rho g \delta z = m_l g$$

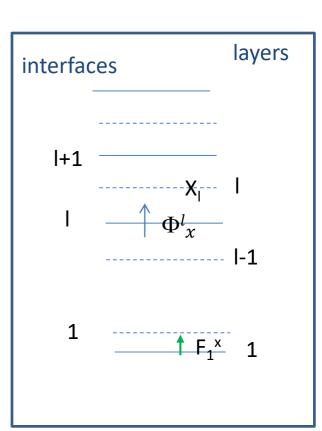
$$\Phi^{l}_{x} = -K_{l} (X_{l}-X_{l-1})$$
 $\frac{\partial X}{\partial t} = -\frac{\partial \Phi}{m_{l}}$

$$m_l \frac{X_l(t+\delta t) - X_l(t)}{\delta t} = \phi_l(t+\delta t) - \phi_{l+1}(t+\delta t)$$

$$m_l \frac{X_l - X_l^0}{\delta t} = \phi_l - \phi_{l+1} \quad \text{with} \quad \begin{array}{l} X_l = X_l(t + \delta t) \\ X_l^0 = X_l(t) \end{array}$$

$$m_l \frac{X_l - X_l^0}{\delta t} = K_{l+1}(X_{l+1} - X_l) - K_l(X_l - X_{l-1})$$

$$-K_{l}X_{l-1} + \left(\frac{m_{l}}{\delta t} + K_{l+1} + K_{l}\right)X_{l} + K_{l+1}X_{l+1} = \frac{m_{l}}{\delta t}X_{l}^{0}$$



Tri-diagonal system that can be solved for the vector X= Enthalpy, specific humidity.wind...

Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$(\delta P_l + R_{l+1}^X + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1}$$
 with $R_l^X = g \delta t K_l$

At the top (I=n, Φ_n =0)

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

At the bottom: (I=1): $m_1 \frac{X_1 - X_1^0}{\delta t} = \Phi_x^1 - \Phi_x^2$

$$m_1 \frac{X_1 - X_1^0}{\delta t} = K_2(X_2 - X_1) - F_1^X$$
$$\left(\delta P_1 + R_1^X\right) X_1 = \delta P_1 \ X_1^0 + R_2^X \ X_2 - g\delta t \underline{F_1^X}$$

With F_1^X : flux of X at the bottom of the first layer (i.e. between the surface and the atmosphere), positive downward.

 $K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l}$

Solving the tridiagonal system

Starting from top:

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

can be written as:

$$X_n = C_n^X + D_n^X X_{n-1}$$

with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$
$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$

with
$$R_l^X = g\delta t K_l$$

Solving the tridiagonal system

$$X_{l+1} = C_{l+1}^X + D_{l+1}^X X_l$$

$$\left(\delta P_l + R_{l+1}^X + R_l^X\right) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1} \quad (2 \le l < n)$$

$$(\delta P_{l} + R_{l+1}^{X} (1 - D_{l+1}^{X}) + R_{l}^{X}) X_{l} = \delta P_{l} X_{l}^{0} + R_{l+1}^{X} C_{l+1}^{X} + R_{l}^{X} X_{l-1}$$
with $R_{l}^{X} = g \delta t K_{l}$

So we obtain by reccurence:

$$X_l = C_l^X + D_l^X X_{l-1} \qquad (2 \le l \le n)$$

with, for $(2 \le l < n)$

depend only on properties in the layers above and the variables at the previous time step.

$$C_l^X = \frac{X_l^0 \delta P_l + R_{l+1}^X C_{l+1}^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$
$$D_l^X = \frac{R_l^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

In LMDZ routine calc_coef

Solving the tridiagonal system

At the bottom of the boundary layer $X_2 = C_2^X + D_2^X X_1$

$$(\delta P_1 + R_2^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g \delta t F_1^X$$

replacing X_2 in the equation above:

$$X_1 = A_1^X + B_1^X . F_1^X . \delta t$$

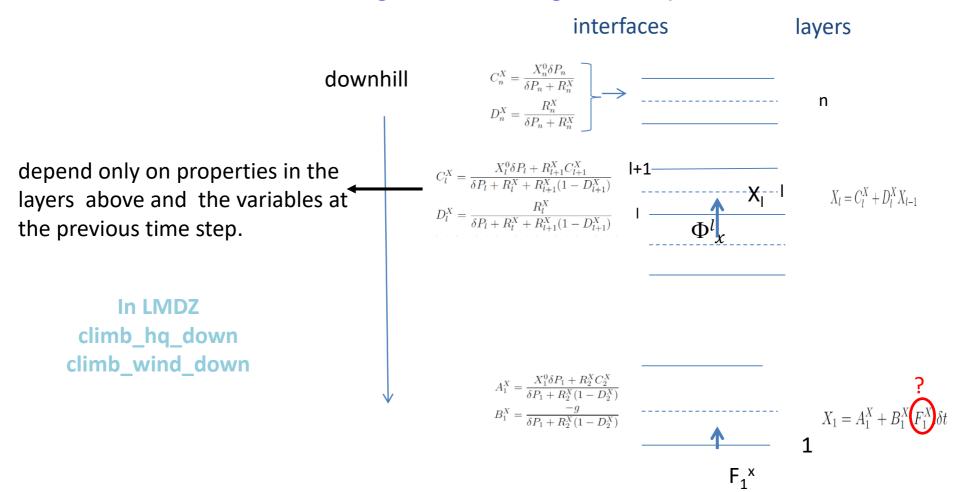
with

$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X (1 - D_2^X)}$$
$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$

$$F_1^{u} = \rho VC_{d,m} (u_1 - u_0)$$

$$u_1 = A_1 + B_1 F_1^{u} \delta t$$

Solving the tridiagonal system



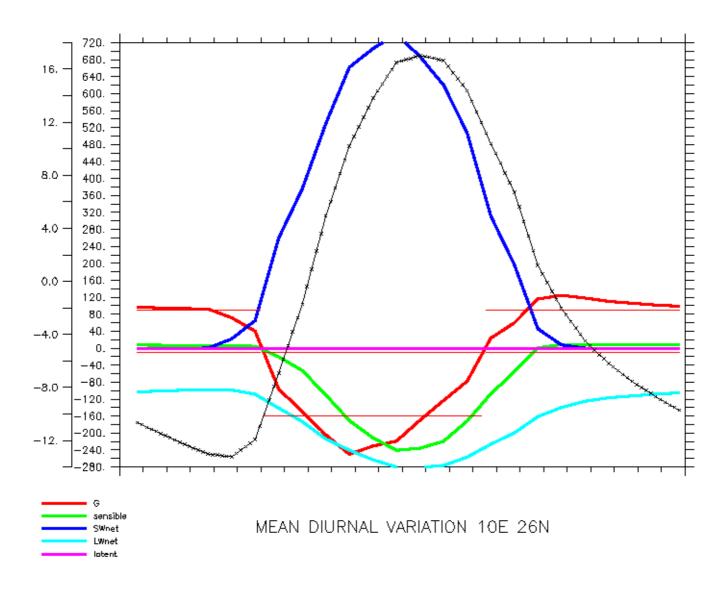
X= wind, enthalpie, specific humidity, tracers

Once F_1^x (flux of water mass, heat between the surface and the atmosphere) is known, the X_i can be computed from the first layer to the top of the PBL

Surface energy budget:

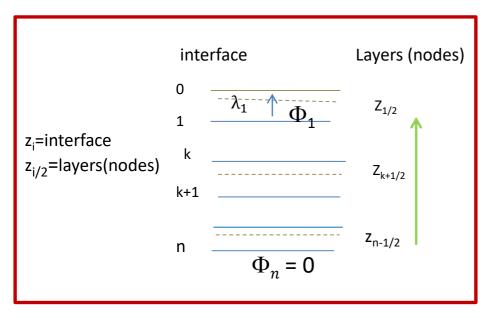
Case of the continental surface

$$SW_{net} + LW_{net} + F + L + \Phi_1 = 0$$



Fast Variations requires an implicit approach to solve the energy budget equations

• Heat conduction : Diffusion equation $C\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (\lambda \frac{\partial T}{\partial z})$



$$\Phi_T = -\lambda \frac{\partial T}{\partial z} \; ;$$

• Bottom: $\Phi = 0$

$$C_{p_{n-1/2}}^{t} \frac{T_{n-1/2}^{t} - T_{n-1/2}}{\delta t} = \frac{1}{z_{N} - z_{N-1}} \left[-\lambda_{n-1} \frac{T_{n-1/2}^{t} - T_{n-3/2}^{t}}{z_{n-1/2} - z_{n-3/2}} \right]$$

Intermediate layers

$$C_{p_{k+1/2}}^{t} \frac{T_{k+1/2}^{t} - T_{k+1/2}^{t}}{\delta t} = \frac{1}{z_{k+1} - z_{k}} \left[\lambda_{k+1} \frac{T_{k+3/2}^{t} - T_{k+1/2}^{t}}{z_{k+3/2} - z_{k+1/2}} - \lambda_{k} \frac{T_{k+1/2}^{t} - T_{k-1/2}^{t}}{z_{k+1/2} - z_{k-1/2}} \right]$$

First layer

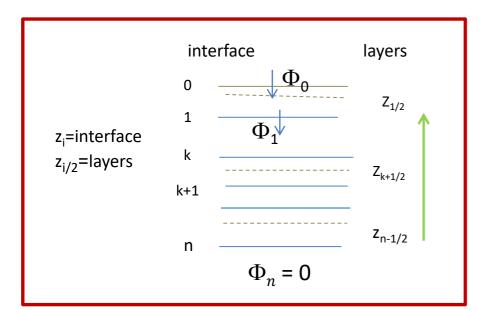
$$T_{3/2}^{t} = \alpha_1^t T_{\frac{1}{2}}^{t} + \beta_1^t$$

Heat conduction : Diffusion equation

We obtain by recurrence (same as for atmosphere)

$$\Phi_T = -\lambda \frac{\partial T}{\partial z} ;$$

$$\frac{\partial T}{\partial t} = -\frac{1}{C} \frac{\partial \Phi_T}{\partial z}$$



First layer

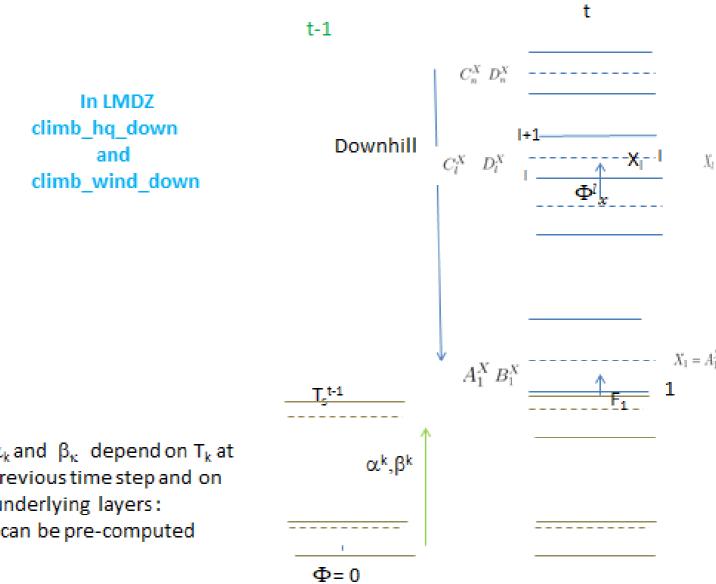
$$T_{3/2}^{t} = \alpha_1^t \ T_{\frac{1}{2}}^{t} + \beta_1^t$$

Intermediate layers

$$T_{k+1/2}^{t} = \alpha_k^t T_{k-1/2}^t + \beta_k^t$$

At t, α_k and β_κ depend on $T_{k1/2}$ at the previous time step they can be computed with a recurrence relationship from one layer to the other.

• **Bottom**:
$$\Phi_n = 0$$
 $T_{n-1/2}^t = \alpha_{n-1}^t T_{n-\frac{3}{2}}^t + \beta_{n-1}^t$



At t α_k and $\,\beta_\kappa\,$ depend on T_k at the previous time step and on the underlying layers: They can be pre-computed

• Top: Continuity between sub-surface and atmosphere + vertical discretization

$$\Phi_1 = \text{Rad} + \sum F^{\downarrow}(T_S^t) - \varepsilon \sigma(T_S^t)^4$$

$$C_{p_{1/2}}^{t} \frac{T_{1/2}^{t} - T_{1/2}}{\delta t} = \frac{1}{z_{1} - z_{0}} \left[\lambda_{1} \frac{T_{3/2}^{t} - T_{1/2}^{t}}{z_{3/2} - z_{1/2}} \right] + Rad + \sum_{s} F^{\downarrow} (T_{S}^{t}) - \varepsilon \sigma (T_{S}^{t})^{4}$$

Top: Continuity between sub-surface and atmosphere

(1)
$$C_{p_{1/2}}^{t} \frac{T_{1/2}^{t} - T_{1/2}}{\delta t} = \frac{1}{z_{1} - z_{0}} \left[\lambda_{1} \frac{T_{3/2}^{t} - T_{1/2}^{t}}{z_{3/2} - z_{1/2}} \right] + \sum_{S} F^{\downarrow} \left(T_{S}^{t} \right) - \epsilon \sigma (T_{S}^{t})^{4}$$

(2)
$$T_{3/2}^t = \alpha_1^t T_{\frac{1}{2}}^t + \beta_1^t$$

 ${\rm T_s}$ is extrapolated as a function of $T_{\!\frac{1}{2}}^t$ taking advantage of (2) (very thin layers, and continuity of the temperature - Hourdin 1993))

$$C'*\frac{T_S^t - T_S^0}{\delta t} = G'* + SW_{net} + LWd + \sum_{s} F^{\downarrow}(T_S^t) - \epsilon\sigma(T_S^t)^4$$

Case of the continental surface

$$\begin{cases} F_{\rm s,H}^t = A^1_H + B^1_H F_{\rm s,H}^t \partial t & \textit{Turbulent diffusion Atmosphere} \\ F_{s,H}^t = \frac{1}{zikt} (H_1^t - H_s^t) & \frac{1}{zikt} = \rho \, |\overrightarrow{v}| \, C_d & \textit{Bulk formulation} \end{cases}$$

$$F_{s,H}^{t} = \frac{1}{zikt}(A_{H}^{1} + B_{H}^{1}.F_{s,H}^{t}\delta t - H_{s}^{t})$$

$$F_{s,H}^t = \frac{1}{zikt} [\frac{(A_H^1 - H_s^{t-\delta t})}{1 - \frac{1}{zikt} B_H^1 \delta t} - \frac{(H_s^t - H_s^{t-\delta t})}{1 - \frac{1}{zikt} B_H^1 \delta t}] \quad \begin{array}{l} \mathsf{C_d}^\mathsf{x} \text{ drag coefficient (Monin Obukhov, constant for in the surface layer)} \\ \text{depends on} \\ \bullet \quad \text{roughness lenghts (gustiness, vegetation),} \end{array}$$

Top boundary condition:

Continuity of the fluxes and the temperature between sub-surface and atmosphere

$$C'*\frac{T_S^t - T_S^0}{\delta t} = G'* + SW_{net} + LW_d + \sum_{s} F^{\downarrow}(T_S^t) - \varepsilon\sigma(T_S^t)^4$$

Solved using the sensitivity of the flux to the surface temperature to calculate the flux at the new time-step

$$F_{s,H}^{t} = sensfl_{old} - sensfl_{sns}(T_{s}^{t} - T_{s}^{t-\delta t})$$

$$\sigma * {T_s^{t - \delta t}}^4 - 4\varepsilon \sigma {T_s^{t - \delta t}}^3 (T_s^t - T_s^{t - \delta t})$$

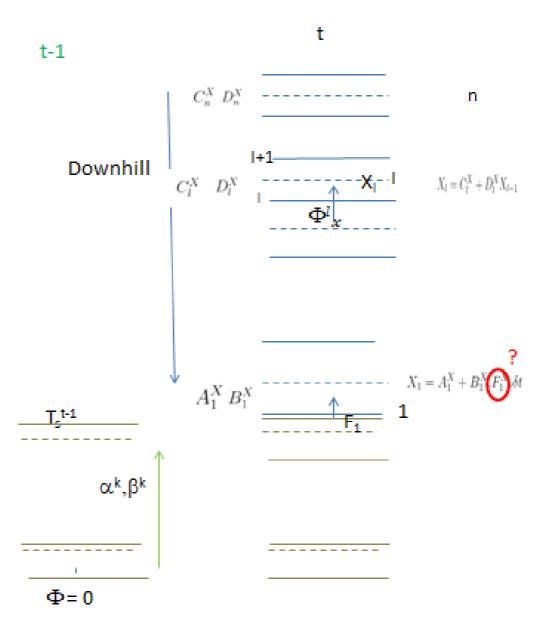
$$T_{\rm S}^{\rm t} = f(SW_{\rm net} + LWd, T_{\rm S}^0, F_{\rm S}^0)$$

In LMDZOR

In LMDZ
climb_hq_down
and
climb_wind_down

At t α_k and β_κ depend on T_k at the previous time step and on the underlying layers: They can be pre-computed

ORCHIDEE (thermosoil)



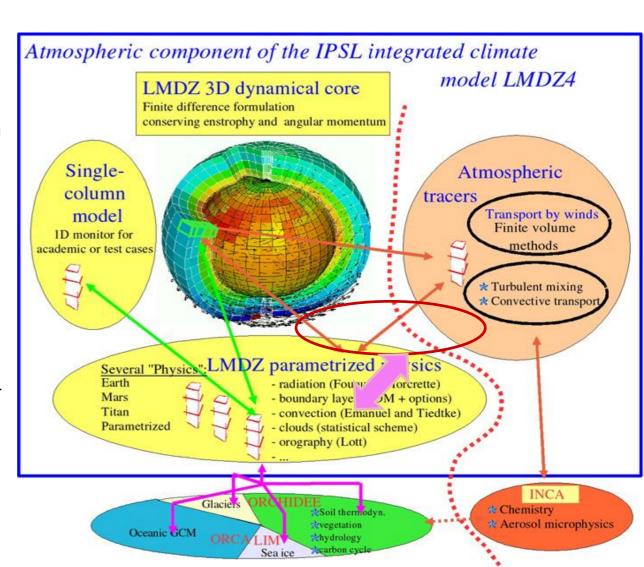
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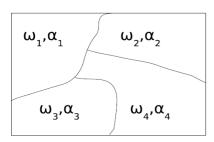


Coupling between atmospheric column(s) and sub-surfaces

Each grid cell is divided into several sub-areas or ``sub-surfaces" of fractions ω_i

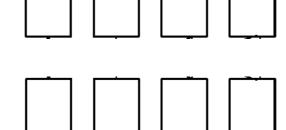
Sub-surfaces

$$\sum_{i} \omega_i = 1$$



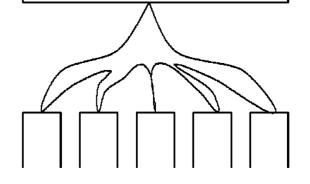
Turbulent flux

One PBL over **each** sub-surface



Radiative flux

One column covers all the subsurface



Each sub surface has to compute F_1 using variables X_1 , A_1 and B_1

The boundary layer tendencies in the atmosphere are mixed between sub-columns (equivalent of averaging the surface flux)

Derivation of local sub-surface **net solar** radiation from grid average net solar radiation

The grid average net flux Ψ_s at surface has been computed for each grid point by the radiative code

We want (1) to conserve energy and (2) to take into account the value of the local albedo αi of the sub-surface.

We compute the downward SW radiation as

$$F^s_{\downarrow} = \frac{\Psi_s}{(1 - \alpha)}$$

with the mean albedo

$$\alpha = \sum_{i} \omega_i \alpha_i$$

For each sub-surface i, the absorbed solar radiation reads:

$$\psi_i^s = (1 - \alpha_i) F_{\perp}^s$$

$$\psi_i^s = \frac{(1 - \alpha_i)}{(1 - \alpha)} \bar{\Psi}^s$$

One may verify that this procedure ensure energy conservation, i.e. $\sum_i \omega_i \psi_i^s = \Psi_s$

$$\sum_{i} \omega_i \psi_i^s = \Psi_s$$

Derivation of local sub-surface **net longwave** radiation from grid average net longwave radiation

The net longwave (LW) radiation $\bar{\Psi}^L$ has been computed by the radiative code for each grid cell. How to split it depending on the sub-surfaces local properties and ensuring energy conservation?

If the downward longwave flux F_{\downarrow} is uniform within each grid, the net LW flux for a sub-surface i may be written as:

$$\psi_i^L(T_i) = \epsilon_i \left(F_{\downarrow} - \sigma T_i^4 \right) \tag{1}$$

where T_i is the surface temperature of sub-surface i and ϵ_i its emissivity. A linearization around the mean temperature \bar{T} gives:

$$\psi_i^L(T_i) \approx \epsilon_i \left(F_{\downarrow} - \sigma \bar{T}^4 \right) - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T})$$
 (2)

To conserve the energy, the following relationship must be true:

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\Psi}^{L} \tag{3}$$

Using Eq. 2 gives

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\epsilon} \left(F_{\downarrow} - \sigma \bar{T}^{4} \right) - 4\sigma \bar{T}^{3} \sum_{i} \omega_{i} \epsilon_{i} \left(T_{i} - \bar{T} \right)$$

$$\tag{4}$$

where $\bar{\epsilon} = \sum_{i} \omega_{i} \epsilon_{i}$ is the mean emissity.

Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

$$\sum_{i} \omega_{i} \psi_{i}^{L} = \bar{\epsilon} \left(F_{\downarrow} - \sigma \bar{T}^{4} \right) - 4\sigma \bar{T}^{3} \sum_{i} \omega_{i} \epsilon_{i} \left(T_{i} - \bar{T} \right)$$

$$\tag{4}$$

where $\bar{\epsilon} = \sum_{i} \omega_{i} \epsilon_{i}$ is the mean emissity. The second term on the right hand side is zero if

$$\bar{T} = \frac{\sum_{i} \omega_{i} \epsilon_{i} T_{i}}{\bar{\epsilon}} \tag{5}$$

To ensure energy conservation, we need in addition to verify:

$$\bar{\Psi}^L = \bar{\epsilon} \left(F_{\downarrow} - \sigma \bar{T}^4 \right) \tag{6}$$

Which is consistent with the definition of the net LW flux at the surface. We rewrite now Eq. 2 as:

$$\psi_i^L(T_i) \approx \frac{\epsilon_i}{\bar{\epsilon}} \bar{\Psi}^L - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T})$$
 (7)

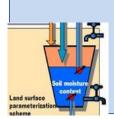
Due to radiative code limitation, in LMDZ, we always must have $\varepsilon_i = 1$ Energy conservation: the radiation is computed by the atmospheric model,

Atmosphere/surface coupling in LMDZOR

LMDZ (phylmd)

Planetary boundary layer and surface modules

(A_q , B_{q_s} A_H , B_{B_s} C_{dh_s} , A_u , B_{u_s} A_v , B_v , C_{dh} , T_1 , q_{1_s} u_1 , v_{1_s} LW_{net} , LW_{down} , Sw_{net}) AcoefH, AcoefQ, BcoefH, BcoefQ cdragh, Iwdown, swnet





(is_ter, ok_veget = n)
 surf_land_bucket

(soil.F90: soil T, heat capacity, conduction,

calcul_flux : sens,flat,tsurf_new

Hydro= water budget (snow, precip, Evap)



Atmosphere/surface coupling in LMDZOR

LMDZ (phylmd)

Planetary boundary layer and surface modules

(
$$A_q$$
 , B_{q_s} A_H , B_{B_s} C_{dh_s} , A_u , B_{u_s} A_v , B_v , C_{dh} , T_1 , q_{1_s} u_1 , v_{1_s} LW_{net_s} LW_{down_s} Sw_{net}) AcoefH, AcoefQ, BcoefH, BcoefQ cdragh, lwdown, swnet

(is_ter, ok_veget = y)
surf_land_orchidee



 LW_{dwn} , SW_{net} , LW_{net} , T_1 , q_1 , $cdrag_h$, u_1 , v_1 A_q , B_q , A_H , B_B , rain, snow)

fluxsens, fluxlat, albedo, ε, tsurf_new, z0

intersurf

ORCHIDEE (sechiba)

petA_orc,petB_orc,peqA_orc,peqB_orc,swet, swnet,lwdown, cdrag

Water and Energy budget (surface and soil)

diffuco (z0, albedo , emissivity)
enerbil fluxsens ,fluxlat, tsurf_new
thermosoil G, ztsol

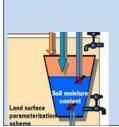
Hydrol: hydrology – diffusion scheme

Atmosphere/surface coupling in LMDZOR

LMDZ (phylmd)

Planetary boundary layer and surface modules

(A_q , B_{q_s} A_H , B_{B_s} C_{dh_s} , A_u , B_{u_s} A_v , B_v , C_{dh} , T_1 , q_{1_s} u_1 , v_{1_s} LW_{net} , LW_{down} , Sw_{net}) AcoefH, AcoefQ, BcoefH, BcoefQ cdragh, lwdown, swnet



(is_ter, ok_veget = n)
 surf land bucket

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calcul_flux : sens,flat,tsurf_new

Hydro= water budget (snow, precip, Evap)

 LW_{dwn} , SW_{net} , LW_{net} , T_1 , q_1 , $cdrag_h$, u_1 , v_1 A_q , B_q , A_H , B_B , rain, snow)

fluxsens, fluxlat, albedo, ε, tsurf_new, z0

intersurf

ORCHIDEE (sechiba)

petA_orc,petB_orc,peqA_orc,peqB_orc,swet, swnet,lwdown, cdrag

Water and Energy budget (surface and soil)

diffuco (z0, albedo , emissivity)
enerbil fluxsens ,fluxlat, tsurf_new
thermosoil G, ztsol

Hydrol: hydrology – diffusion scheme

In subroutine PHYSIQ

loop over time steps

Call tree

CALL change_srf_frac : Update fraction of the sub-surfaces (pctsrf)

• • • •

CALL pbl_surface Main subroutine for the interface with surface

Calculate net radiation at sub-surface

Loop over the sub-surfaces nsrf

Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface in not zero)

CALL cdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)

CALL coef_diff_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)

CALL climb_hq_down downhill for enthalpy H and humidity Q

CALL climb_wind_down downhill for wind (U and V)

CALL surface models for the various surface types: surf_land, surf_landice, surf_ocean or surf_seaice.

Each surface model computes:

- evaporation, latent heat flux, sensible heat flux, momentum
- surface temperature, albedo (emissivity), roughness lengths

CALL climb_hq_up: compute new values of enthalpy H and humidity Q

CALL climb_wind_up : compute new values of wind (U and V)

Uncompress variables: (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)

Cumulate in global variables after weighting by sub-surface fractions Surface diagnostics: (T, q, wind are evaluated at a reference level (2m) owing to an interpolation scheme based on the MO laws).

End Loop over the sub-surfaces

Calculate the mean values over all sub-surfaces for some variables

End pbl-surface

THANK YOU FOR YOUR ATTENTION

- Technical note: Description of the interface with the surface and the computation of the turbulent diffusion in LMDZ (J.L.Dufresne)
- Thèse F. Hourdin 1993 (section 3.3.3 and annexes)
- Wang F., F. Cheruy, J.L. Dufresne, 2016: The improvement of soil thermodynamics and its effects on land surface meteorology in the IPSL climate model. Geosci. Model Dev., 9, 363–381, 2016 www.geosci-modeldev.net/9/363/2016/

Case of the continental surface

Surface energy budget

$$SW_{net} + LW_{net} + F + L + \Phi_0 = 0$$

$$L = \beta \rho VC_d (q_1 - q_s(T_s))$$

$$SW_{net} + LW_d - \epsilon \sigma T_s^4 + F + L + \Phi_0 = 0$$

$$E = \rho VC_d (T_1 - T_s)$$

$$depends on Ts$$

Heat conduction in the soil: diffusion equation :

$$\Phi_T = -\lambda \frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = -\frac{1}{c} \frac{\partial \Phi_T}{\partial z}$$

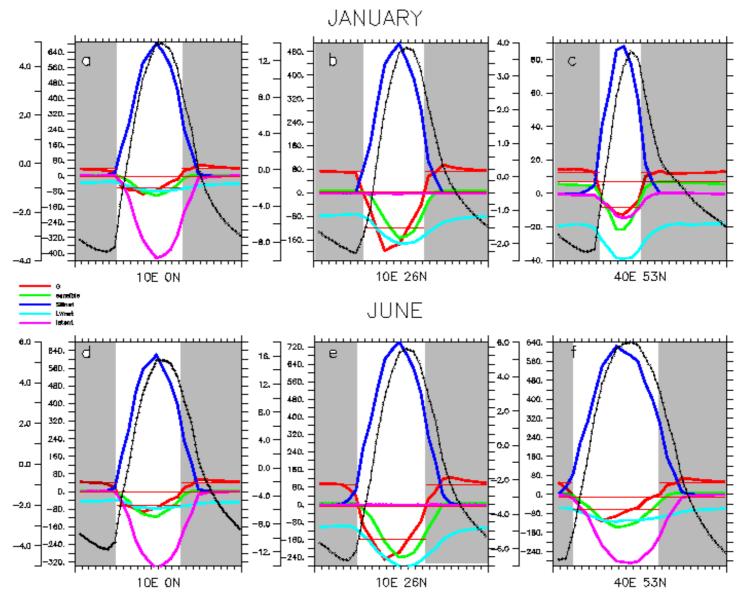
Boundary conditions:

- ✓ bottom: $\Phi = 0$
- ✓ top: Continuity of the fluxes and the temperature between sub-surface and atmosphere

Surface energy budget:

Case of the continental surface

$$SW_{net} + LW_{net} + F + L + \Phi_0 = 0$$



Fast Variations requires an implicit approach to solve the energy budget equations

Cheruy et al. 2019