## LMDZ

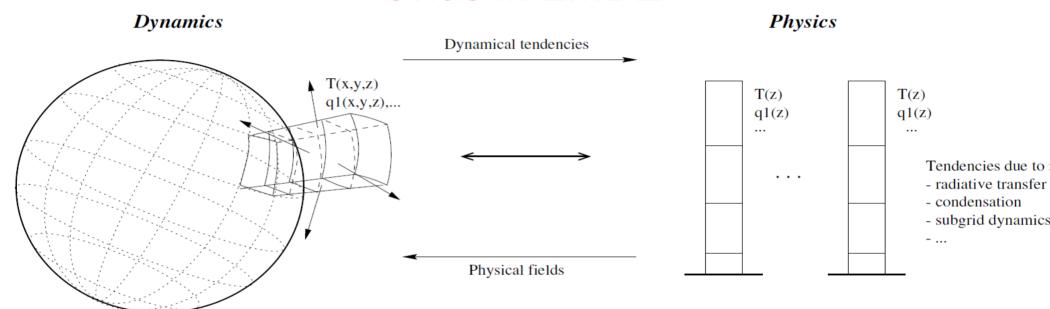
Dynamics/physics organization,
Grids,
Time stepping,
Dissipation...

LMDZ courses, December 12, 2024

### Overview of course topics

- Grids:
  - Horizontal grids in the physics & dynamics
  - Vertical discretization
- Time marching:
  - Generalities about time marching schemes
  - What is used in LMDZ
  - Longitudinal polar filter
- Lateral diffusion and sponge layer:
  - Energy cascade
  - Illustrative example of diffusion
  - Sponge layer near model top

#### Grids in LMDZ



#### Separation between physics and dynamics:

- "dynamics": solving the GFD equations on the sphere; usually with the assumption of a hydrostatic balance and thin layer approximation. Valid for all terrestrial planets.
- "physics": (planet-specific) local processes, local to individual atmospheric columns.

## Horizontal grids in LMDZ

IM

#### Grid dimensions specified when compiling LMDZ:

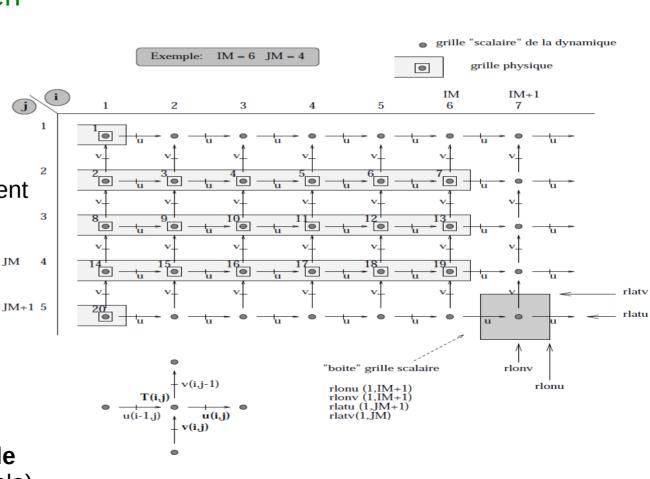
makelmdz fcm -d iimxjjmxllm ...

#### In the dynamics:

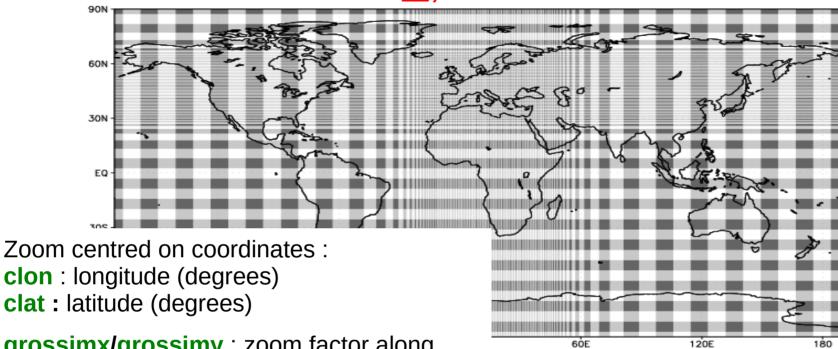
- Staggered grids: u, v and scalars (temperature, tracers) are on different meshes
- Global lonxlat grids with redundant grid points
  - at the poles
  - in longitude

#### In the physics:

- Collocated variables
- No global lonxlat horizontal grid, columns are labelled using a single index (from North Pole to South Pole)



#### LMDZ, Z for Zoom



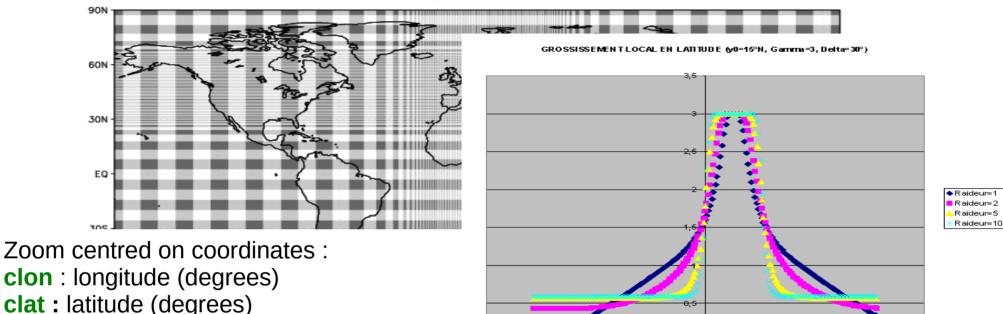
grossimx/grossimy: zoom factor along

x/y directions (i.e. lon/lat)

Computed as the ratio of the smallest mesh (I.e. in the zoom), compared to the mesh size for a global regular grid with the same total number of points.

dzoomx/dzoomy: fraction of the grid containing the zoomed area: dzoom\*360° by dzoomy\*180°

#### LMDZ, Z for Zoom



0.5

Latitude (radians)

grossimx/grossimy : zoom factor along
x/v directions (i.e. lon/lat)

Computed as the ratio of the smallest mesh (I.e. in the zoom), compared to the mesh size for a global regular grid with the same total number of points.

**taux/tauy**: steepness of the transition between inner zoom and outer zoom meshes (typically one tries to avoid sharp transitions; tau ~ 3 is a reasonable value)

#### $\frac{\partial u}{\partial u} = \frac{\partial u}{\partial u} + \frac{u_{analyse} - u}{\partial u}$ $\frac{\partial t}{\partial t} = \frac{\partial t}{\partial t}_{GCM}$ $\frac{\partial v}{\partial v} = \frac{\partial v}{\partial v} + \frac{v_{analyse} - v}{\sigma}$ at at GCM

Nudging towards analyses or reanalyses with chosen time constants

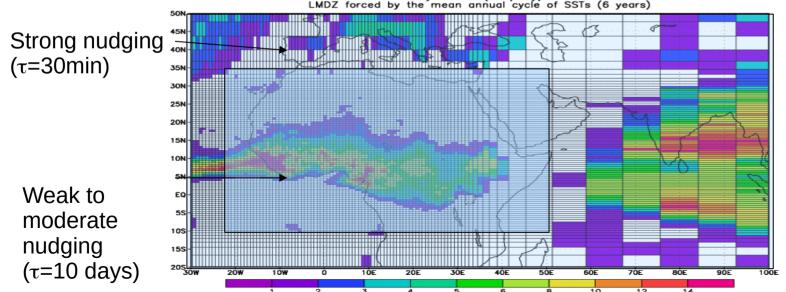
## Nudging in LMDZ

(nudge = "guide" in French)

#### **Example of nudging parameters:**

ok quide = y $guide_T = n$ ,  $guide_p = n$ ,  $guide_q = n$ guide u = y, guide v = y $tau_min_u = 0.0208333 (days)$ tau max u = 10 (days)  $tau_min_v = 0.0208333 (days)$ July rainfall (mm/day) tau\_max\_v = 10 (days)

LMDZ forced by the mean annual cycle of SSTs (6 years)

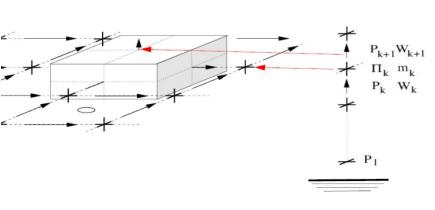


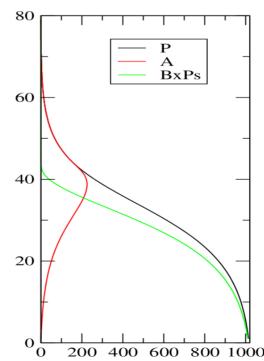
#### Vertical discretization in LMDZ

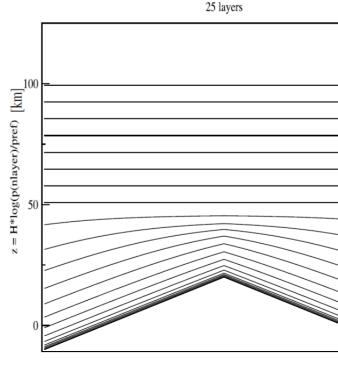
Model levels are hybrid sigma-pressure levels:

P(level,time) = ap(level) + bp(level). Ps (time) hybrid coordinates ap(k) and bp(k) are fixed for a given model run Surface pressure Ps(t) varies during the run

- Near the surface ap ~ 0
   => bp(k) ~ P/Ps
- At high altitudes, bp  $\sim 0$







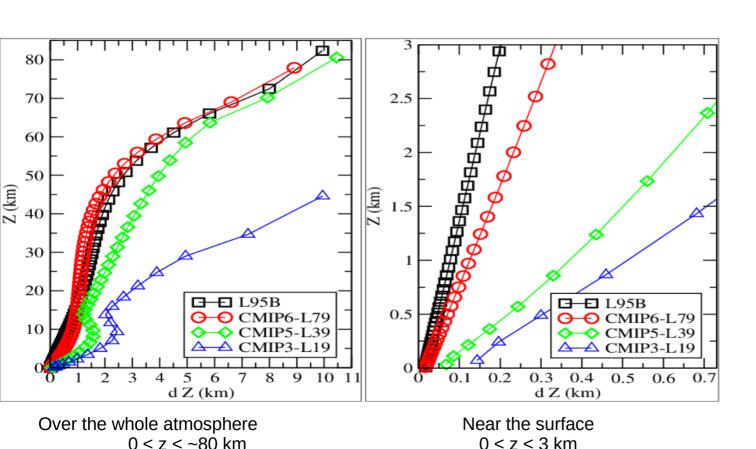
#### Vertical discretization in LMDZ

Setting model levels via the def files (also a function of number of vertical levels):
 vert\_sampling = strato\_custom : customable (via other parameters in .def file; see next slide) discretization for stratospheric extensions.
 Multiple other possibilities from this "default":

=> Check out routine dyn3d common/disvert.F90

#### Vertical discretization in LMDZ

Illustration of typical altitudes and layer thickness of L19/L39/L79/L95 grids



"Standard" CMIP6
L79 settings
(see DefLists/vert\_L79.def)

vert\_sampling=strato\_custom

vert\_scale=7. vert\_dzmin=0.017 vert\_dzlow=1. vert\_z0low=8.7 vert\_dzmid=2. vert\_z0mid=70. vert\_h\_mid=20. vert\_dzhig=11. vert\_z0hig=75. vert\_h hig=20.

## Questions?

• The big picture: you want to solve

$$\frac{df(t)}{dt} = R(f,t)$$

$$f(t=0) = f_0$$

from a known initial condition at time t=0 to time t=...

• So it is all about using a time marching scheme, built on Taylor expansion for evaluation of the time derivative, and choosing at which time level  $t=n.\delta t$  the right hand side term R[f(t),t] is to be evaluated:

$$f(t_0 + \delta t) = f(t_0) + \frac{\delta t}{1!} f'(t_0) + \frac{(\delta t)^2}{2!} f''(t_0) + \dots$$

• Explicit Euler scheme (1st order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f,t) \simeq R(f(t_n), t_n)$$

• Implicit Euler scheme (1st order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f,t) \simeq R(f(t_{n+1}), t_{n+1})$$

• Crank-Nicholson scheme (2<sup>nd</sup> order in time):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$R(f,t) \simeq \frac{R(f(t_{n+1}), t_{n+1}) + R(f(t_n), t_n)}{2}$$

• Matsuno scheme: a predictor-corrector (Euler explicit-Euler Implicit) scheme (1st order):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_n}{\delta t}$$

$$f^p(t_{n+1}) = f(t_n) + \delta t \cdot R(f(t_n), t_n)$$

$$R(f, t) \simeq R(f^p(t_{n+1}), t_{n+1})$$

• Leapfrog scheme: use encompassing time steps to evaluate the derivative (2<sup>nd</sup> order):

$$\frac{df(t)}{dt} \simeq \frac{f_{n+1} - f_{n-1}}{2\delta t}$$

$$R(f,t) \simeq R(f(t_n), t_n)$$

• Illustrative example, on a decay equation (known solution!)

$$\frac{dq(t)}{dt} = -\frac{1}{\tau}q(t) \longrightarrow q(t) = q_0 e^{-\frac{t}{\tau}}$$

Building Euler explicit (E) & implicit (I) schemes:

$$\frac{dq(t)}{dt} = -\frac{1}{\tau}q(t)$$

$$\Rightarrow \frac{q^{n+1} - q^n}{\delta t} \simeq -\frac{1}{\tau}q^n \quad \text{(E.E.)}$$

$$\Rightarrow q^{n+1} - q^n = -\frac{\delta t}{\tau}q^n \quad \text{(E.E.)}$$

$$\Rightarrow q^{n+1} - q^n = \left[1 - \frac{\delta t}{\tau}\right]q^n \quad \Rightarrow \frac{q^{n+1} - q^n}{\delta t} \simeq -\frac{1}{\tau}q^{n+1} \quad \text{(E.I.)}$$

$$\Rightarrow q^{n+1} - q^n = -\frac{\delta t}{\tau}q^{n+1} \quad \Rightarrow q^{n+1} - q^n = -\frac{\delta t}{\tau}q^{n+1} \quad \Rightarrow q^{n+1} = q^n \quad \Rightarrow q^{n+1} = q^n \quad \Rightarrow q^{n+1} = \frac{1}{1 + \frac{\delta t}{\tau}}q^n$$

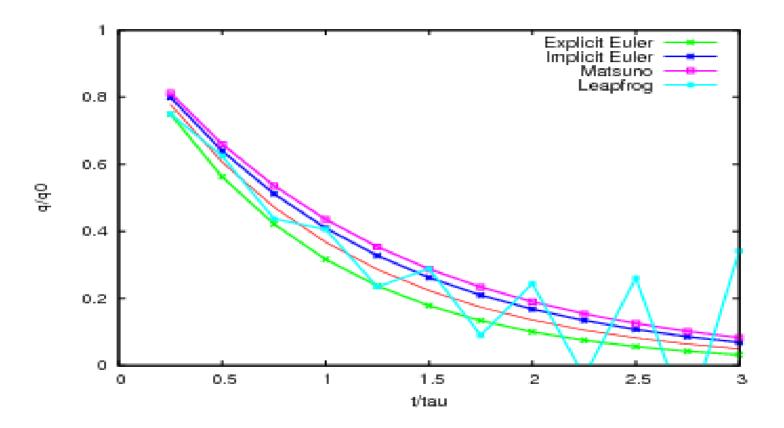
Illustrative example, on a decay equation

$$\frac{dq(t)}{dt} = -\frac{1}{\tau}q(t) \longrightarrow q(t) = q_0 e^{-\frac{t}{\tau}}$$

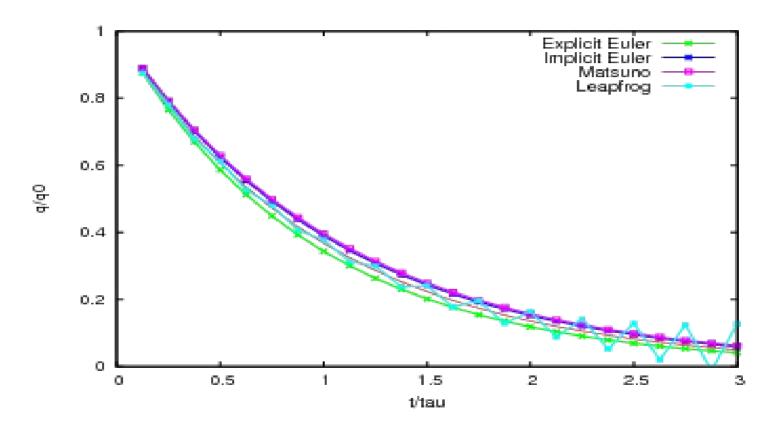
Resulting integration schemes:

EE: 
$$q^{n+1} = \left[1 - \frac{\delta t}{\tau}\right] q^n$$
  
EI:  $q^{n+1} = \left[\frac{1}{1 + \delta t/\tau}\right] q^n$   
CN:  $q^{n+1} = \left[\frac{1 - \delta t/(2\tau)}{1 + \delta t/(2\tau)}\right] q^n$ 

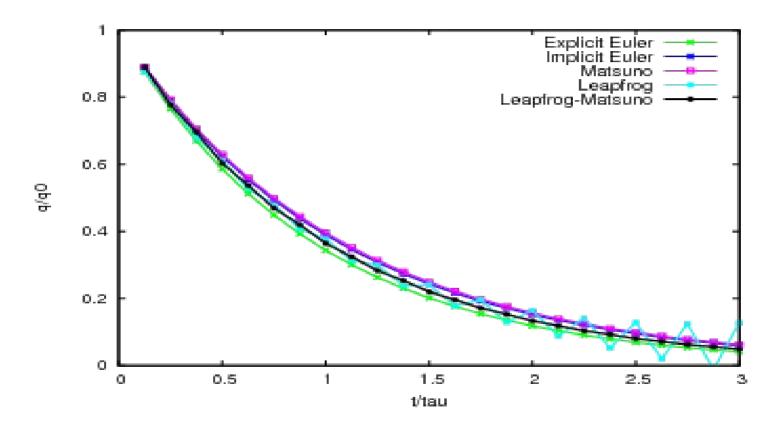
Stability requirement (CFL) for EE: dt/tau < 2</li>



• 4 integration steps per unit of t/tau



• 8 integration steps per unit of t/tau



8 integration steps per unit of t/tau

## Time marching in LMDZ

Time splitting between physics/dynamics/dissipation:

$$\frac{\partial \psi}{\partial t} = Dyn(\psi) + Phy(\psi) + Dissip(\psi)$$

- Dynamics: Leapfrog-Matsuno scheme
   Using day\_step dynamical steps per day
   Leapfrog steps with a Mastuno step every iperiod step
- Physics: Explicit Euler
   Every iphysiq dynamical steps (multiple of iperiod)
- Dissipation: Explicit Euler
   Every dissip\_period dynamical steps (multiple of iperiod)

## Side note about explicit or implicit time marching schemes

Even when solving linear spatio-temporal boundary-value problems, e.g.:  $\frac{dA}{dt} = \kappa \frac{\partial^2 A}{\partial x^2}$ 

The explicit Euler approach leads to a straightforward expression for grid point values (but with stability constraints):

$$\frac{A_i^{k+1} - A_i^k}{\delta t} = \frac{\kappa}{h^2} \left[ A_{i-1}^k - 2A_i^k + A_{i+1}^k \right]$$

Whereas the implicit Euler approach leads to a (tridiagonal) system of equations to solve:

$$\frac{A_i^{k+1} - A_i^k}{\delta t} = \frac{\kappa}{h^2} \left[ A_{i-1}^{k+1} - 2A_i^{k+1} + A_{i+1}^{k+1} \right]$$

=> requires more computations, but may be necessary if time-stepping constrains require using large time steps.

# Side note about tridiagonal system solving

When needing to solve a tridiagonal system of the form:

T.x=y , T tridiagonal matrix, x & y vectors

Rather than invert T (costly!) to generate  $T^{-1}$  (dense matrix) and then compute  $x=T^{-1}$ .y (matrix-vector product)

Use the LU decomposition (Gaussian elimination) of T to split the problem into two very simple sub-problems:

- 1) L.U=T, L and U are bidiagonal (lower/upper) matrices
- 2) Solve L.z=y for vector z (forward substitution step)
- 3) Solve U.x=z for vector x (backward substitution step)

# Side note about incorporating boundary conditions (BC)

$$\frac{\partial f(x,t)}{\partial t} = R(f,x,t) \text{ with BC: } f(x_B,t) = B(x_B,t)$$

- In practice one needs to **treat specifically** the spatial boundary conditions, which could be either imposed on the value (Dirichlet) or its derivative (Neumann) or a mix of the two (Robin) or ...
- The choice of using an implicit or explicit representation also impacts the incorporation of the boundary conditions (along spatial directions)

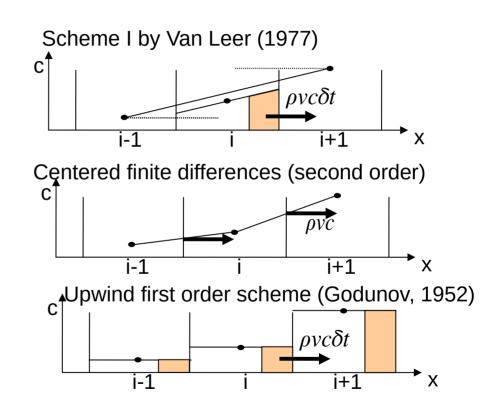
Explicit (B<sub>n</sub> known) => 
$$\frac{f_{n+1}-f_n}{\delta t} \simeq R_n \text{ with BC: } B_n$$

Implicit (B<sub>n+1</sub> unknown) => 
$$\frac{f_{n+1}-f_n}{\delta t} \simeq R_{n+1}$$
 with BC: B<sub>n+1</sub>

## Tracer advection in LMDZ

Use of the Van Leer I scheme (1977), a second order finite volume scheme with slope limiters (e.g. MUSCL, MINMOD) (Hourdin et Armengaud, 1999).

Guaranties of fundamental physical properties of transport: conservation of the total quantity, positivity, monotony, non amplification of extrema, weak numerical diffusion



 CFL requirement, for an advection velocity Umax : Umax.(dt/dx) = cte , with cte ~ 0(1)

## Tracer advection in LMDZ

• In practice: Tracer names and various properties are set in the tracer.def file. e.g.: &version=1.0 &lmdz default type=tracer phases=g hadv=10 vadv=10 parent=air H2O phases=q hadv=14 vadv=14 H2O phases=I havd=10 vadv=10 H2O phases=s hadv=10 vadv=10

Rn

Pb

- Scheme "10": Van Leer scheme
- Scheme "14": Dedicated modified scheme for water vapor
- Other (experimental) schemes are coded; see <a href="https://doi.org/10.150/doi

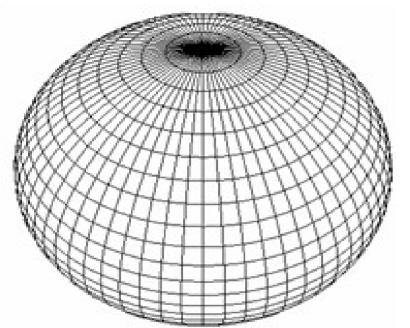
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- phases: gas/liquid/solid for a given tracer
- parent: air or another tracer (for isotopes)+ possibility of "tagging" see https://lmdz-forge.lmd.jussieu.fr/mediawiki/LMDZPedia

## Questions?

## The longitudinal polar filter



- A lon-lat grid implies that the meshes tighten dramatically as the pole is approached.
- CFL conditions there would dictate using an extremely small time step for the time marching scheme.

- Longitudinal (Fourier) filtering, removing high spatial frequencies, is used to enforce that resolved features are at the level of those at ~60° (where longitudinal resolution is half of that at the equator).
- In addition near the poles there is some longitudinal grouping of meshes (applied to the divergence of air transport) by bunches of 2ngroup (typically ngroup=3) which implies that the number of points along longitudes of the GCM must be a multiple of 2ngroup!

## Energy spectra and lateral dissipation

 Observations (Nastrom & Gage 1985, Lindborg 1999) collected over length scales from a few to thousands of km display a characteristic energy cascade (from Skamarock, 2004).

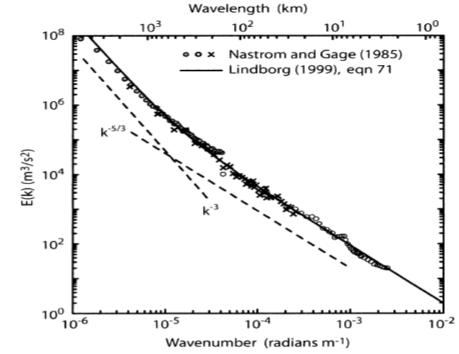


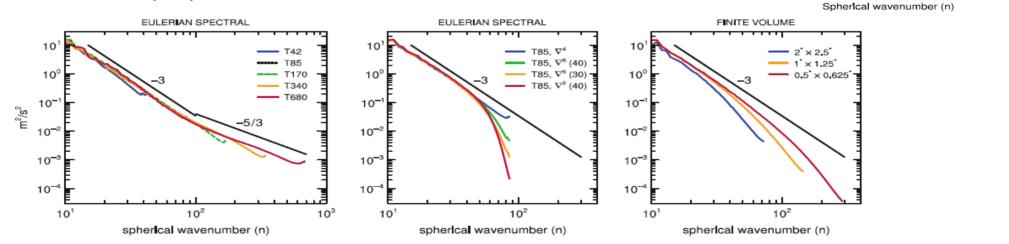
FIG. 1. Nastrom and Gage (1985) spectrum derived from the GASP aircraft observations (symbols) and the Lindborg (1999) functional fit to the MOZAIC aircraft observations.

 In order to fulfil the observed energy cascade from resolved scales to unresolved scales in GCMs, a dissipation term is added:

$$Dissip(\psi) = \frac{(-1)^{q+1}}{\tau} \nabla^{2q} \psi$$

# Lateral dissipation in GCMs as a tool to pin the energy cascade

Figures from Numerical Techniques for Global Atmospheric Models, Lauritzen et al. (eds), 2010



10<sup>2</sup>

10°

10<sup>-3</sup>

10-4

100

FV, no div damping FV. 2nd-order div

EUL T85, 4th-order hyper-diffusion

 $10^{2}$ 

FV. 4th-order div

Kinetic energy (m²/s²)

Fig. 13.4 250 hPa kinetic energy spectra as a function of the spherical wavenumber (n) in aquaplanet simulations from (left) CAM Eulerian spectral dynamical core with  $\nabla^4$  diffusion for different resolutions, (center) T85L26 Eulerian spectral dynamical with  $\nabla^4$ ,  $\nabla^6$  and  $\nabla^8$  diffusion, and (right) CAM Finite Volume (FV) dynamical core for different  $lat \times lon$  resolutions in degrees and 26 levels

## Illustrative example of dissipation

• Simple 1D diffusion equation toy model:

$$\frac{dA}{dt} = \nu \frac{\partial^2 A}{\partial x^2}$$

Von Neumann (Fourier mode) analysis

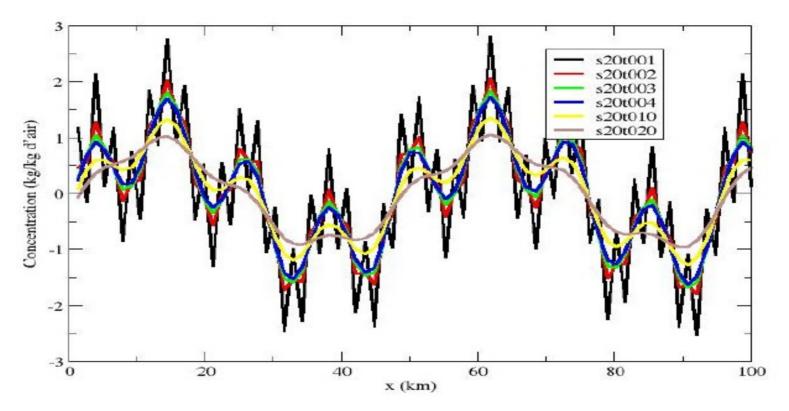
$$A_k(t) = a_k(t).sin(kx)$$

• Explicit Euler time marching (with stability condition!):

$$a_k^{n+1} = \left(1 - \nu k^2 \Delta t\right) a_k^n$$

Note that mode damping is stronger for large k

#### Illustrative example of dissipation



Temporal evolution, from an initial condition consisting of 2 sine modes and an extreme (2 grid points wavelength) "numerical mode"

## Controlling dissipation in LMDZ

• Parameters in file gcm.def:

dissip\_period: Apply dissipation every dissip\_period dynamical steps (or specify 0 to let model pick an appropriate value)

**nitergdiv**: number of iterations on velocity dissipation operator grad.div **nitergrot**: number of iterations on velocity dissipation operator grad.rot

**niterh**: number of iterations on temperature dissipation operator div.grad

**Usual values:** nitergdiv=1, nitergrot=2, niterh=2

**tetagdiv**: dissipation time scale (s) for smallest wavelength for u,v (grad.div component)

**tetagrot**: dissipation time scale (s) for smallest wavelength for u,v (grad.rot component)

**tetatemp**: dissipation time scale (s) for smallest wavelength for potential temperature (div.grad)

values depend on horizontal resolution

### Controlling dissipation in LMDZ

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**tetagdiv**: dissipation time scale (s) for smallest wavelength for u,v (grad.div component)

**tetagrot**: dissipation time scale (s) for smallest wavelength for u,v (grad.rot component)

**tetatemp**: dissipation time scale (s) for smallest wavelength for potential temperature (div.grad)

optimal teta values depend on horizontal resolution

 Moreover there is a multiplicative factor for the dissipation coefficient, which increases with model levels (see dyn3d\_common/inidissip.F90), which can be controlled by flag "vert\_prof\_dissip"

#### The sponge layer

- In addition to lateral dissipation, it is necessary to damp vertically propagating waves (non-physically reflected downward from model top).
- The sponge layer is limited to topmost layers (usually 4) and added during the dissipation step.
- Sponge modes and parameters (gcm.def):

**iflag\_top\_bound**: 0 for no sponge, 1 for sponge over 4 topmost layers, 2 for sponge from top to 100 times topmost layer pressure

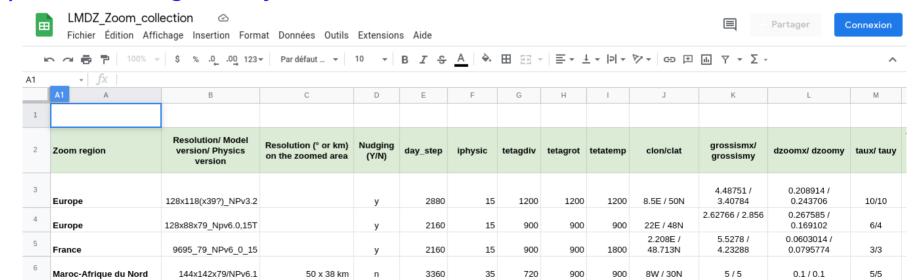
**mode\_top\_bound**: 0 for no relaxation, 1 to relax u,v to zero, 2 to relax u,v to their zonal mean, 3 to relax u,v and potential temperature to their zonal mean.

tau\_top\_bound: inverse of characteristic time scale at the topmost layer (halved at each successive descending layer)

## Where to get typical values of the run.def/gcm.def parameters

- Check out the various examples of gcm.def\_\* files located in the LMDZ/DefLists subdirectory
- Some examples of specific cases of Zoomed simulation setups are detailed in a collaborative document; get the link by searching "Zoom collection" on LMDZPedia:

https://lmdz-forge.lmd.jussieu.fr/mediawiki/LMDZPedia



#### Some rules of thumb for run.def parameters

Time steps in LMDZ:

```
dynamical time steps: dtvr = daysec / day_step
physics time step: dtphys = iphysiq * dtvr
dissipation time step: dtdiss = dissip_period * dtvr
tracer advection time step: dtvrtrac = iapp_trac * dtvr
```

Constraints to be aware of:
 dtvr limited by CFL for waves: Cmax.dt < min(dx,dy)</p>
 dtrtrac limited by advection CFL: Umax.dt < min(dx,dy)</p>
 iphysiq, dtvrtrac, dissip period should be multiples of iperiod

#### Some rules of thumb for run.def parameters

 Constraints to be aware of (continued): dissipation time step should be much smaller than dissipation timescales:

dtdiss << tetatgdiv, tetagrot, tetatemp

Changing time step with resolution on a regular grid:
 day\_step(max(iim,jjm)=N) ~ day\_step(max(iim,jjm)=M) \* M/N

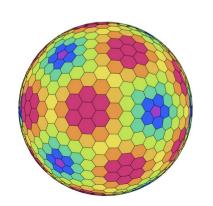
Time step for a zoomed simulation, compared to regular grid:
 day\_step(zoom) ~ day\_step(regular)\*max(grossismx,grossismy)

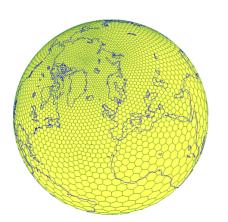
#### An icosahedral grid...

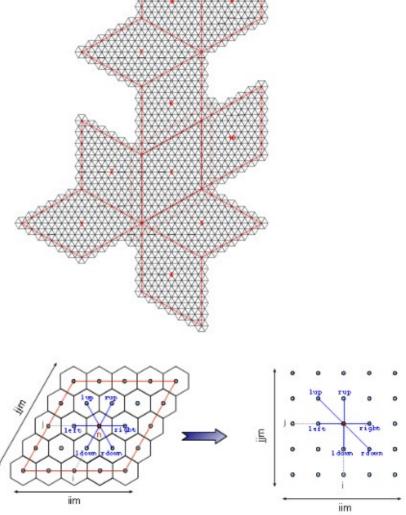
- Cells are mostly (irregular) hexagons, some pentagons
- No zonal filters => more parallelism
- Makes other things trickier!

.. but a similar "user experience".

Especially, files are read/written by XIOS which can interpolate between DYNAMICO and lon-lat mesh. Except for a good reason, **input and output fields are longitude-latitude** 







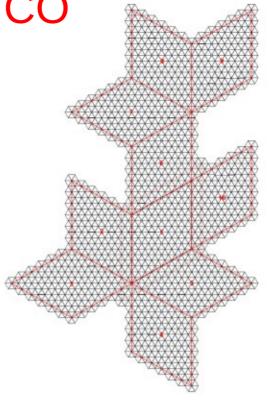
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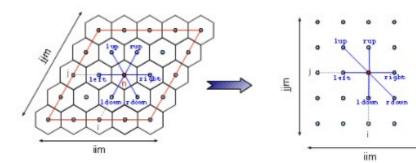
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#### Horizontal resolution

- Controlled by a single parameter "nbp"
- 10 x nbp x nbp grid points (atmospheric columns)
- nbp=40 => about 2 degree resolution (dx ~ 220km)
- dx is divided by 2 when nbp doubles
- maximum dynamics time step:  $dt \sim dx/c$ ,  $c\sim400$  m/s





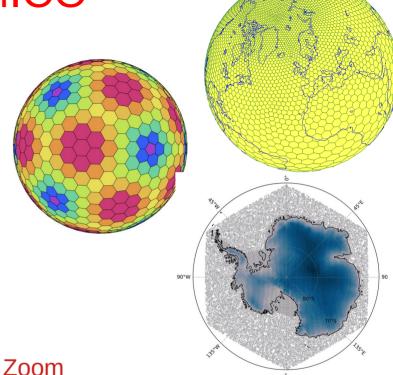
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- "polar" zoom via Schmidt transform
- soon (?) via variable-resolution meshing

#### Limited Area-Model

- metric type = icosa area
- set center (lon, lat) and radius
- dx = radius/nbp
- Needs other parameters, and input files

#### Transport schemes

- Scheme similar to lon-lat Van Leer
- Transport time step can be a small multiple of dynamics time step (itau\_adv)

#### Horizontal dissipation

- (bi-)laplacian, exactly as lon-lat LMDZ
- Coefficients tuned for a certain resolution

#### Sponge layer

- Relaxation to zero: simple but breaks total angular momentum conservation
- Relaxation to zonal mean: implemented (via XIOS) by expensive...
- Other options currently under consideration (relaxation to temporal mean)

#### Nudging

implemented

#### (incomplete) doc of DYNAMICO parameters/flags:

https://gitlab.in2p3.fr/ipsl/projets/dynamico/dynamico/-/blob/master/PARAMETERS.md?ref\_type=heads

#### Search for DYNAMICO on LMDZPedia!:

https://lmdz-forge.lmd.jussieu.fr/mediawiki/LMDZPedia/index.php/Sp%C3%A9cial:Recherche?search=dynamico&fulltext =Recherche+en+texte+int%C3%A9gral&fulltext=Search