The surface-atmosphere interactions

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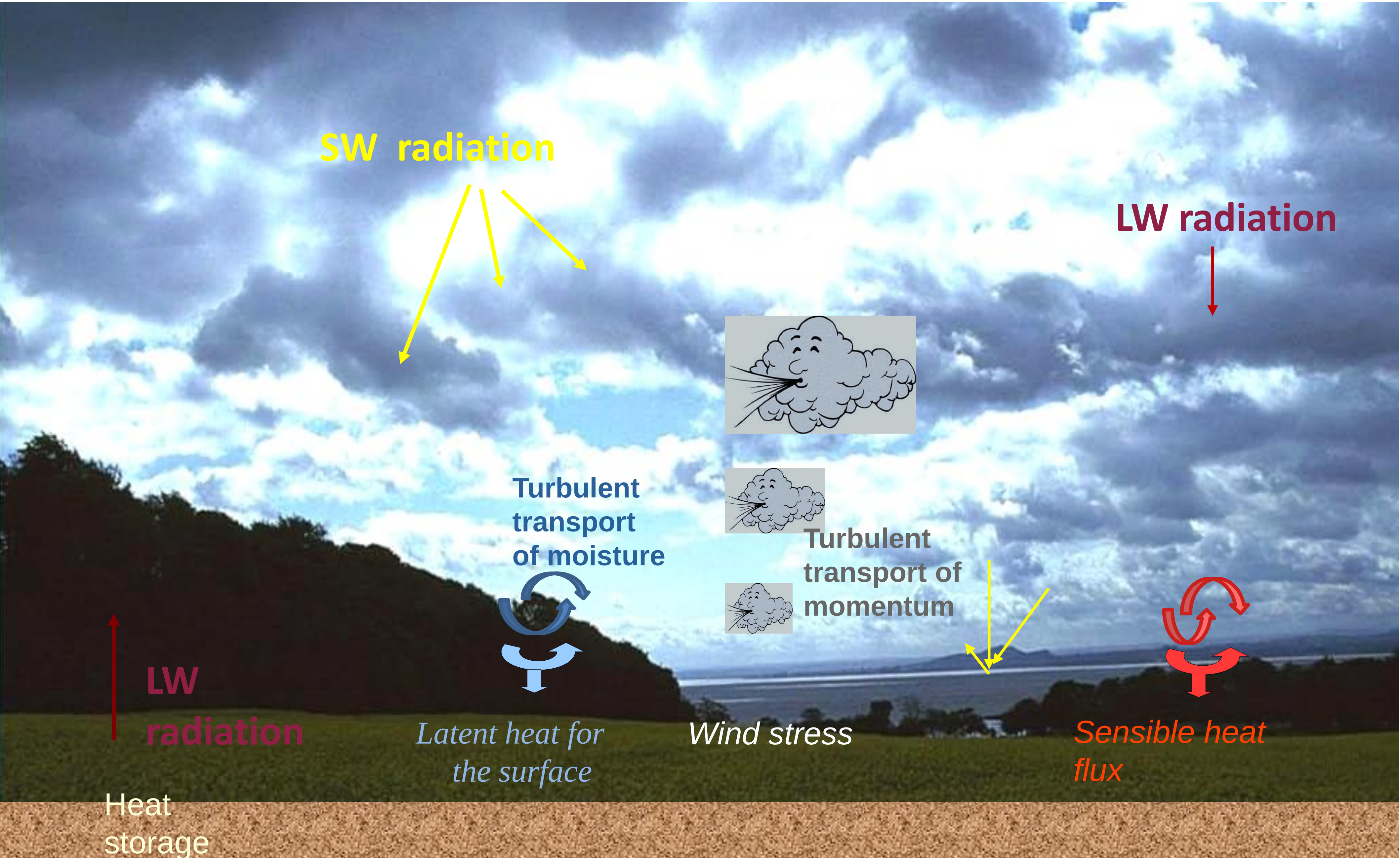
1. Introduction, process involved
2. Radiation over sub-surfaces
3. Turbulent transport
4. Surface energy budget and heat diffusion into the soil
5. Fully coupled surface – atmosphere system
6. Some illustration of the coupling
7. Call tree in LMDZ
8. Take home messages


SW radiation
LW radiation
Turbulent transport of moisture
Turbulent transport of momentum
LW radiation
Latent heat for the surface
Wind stress
Sensible heat flux
Heat storage

***Radiation** at the surface*

depends on mean surface properties (albedo, emissivity)

Turbulent diffusion

depends on local sub-grid properties (roughness)

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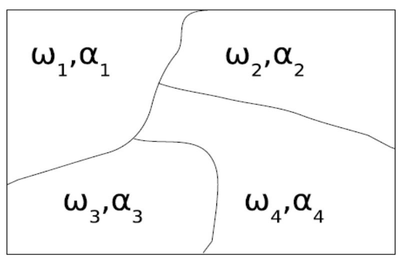
Radiation *at the surface* depends on mean surface properties (albedo, emissivity)

In LMDZ : 4 sub-surfaces : land, land-ice, ocean, sea-ice

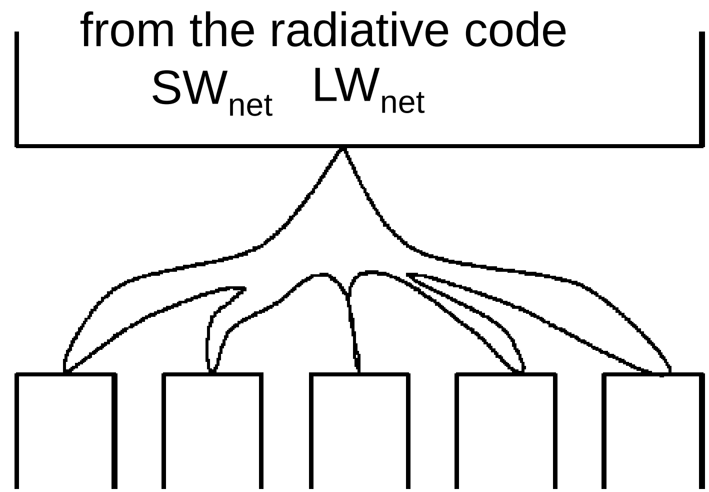
Split the the SW radiation on sub-surfaces

Each grid cell is divided into several sub-areas or ``sub-surfaces" of fractions

$$\sum_i \omega_i = 1$$



One atmospheric column
covers all the sub-surface



$\alpha_{oce}, \epsilon_{oce}$ ..... $\alpha_{land}, \epsilon_{land}$ .....

The grid average net flux SW_{net} has been computed previously by the radiative code

We need to (1) conserve energy and (2) take into account the value α_i of the local albedo of the sub-surface i

$$SW_{dn} = \frac{SW_{net}}{(1-\alpha)} \qquad \alpha = \sum_i \omega_i \alpha_i$$

$$SW^i_{net} = (1-\alpha_i) SW_{dn} = \frac{(1-\alpha_i)}{(1-\alpha)} SW_{net}$$

$$\boxed{\sum_i \omega_i SW^i_{net} = SW_{net}}$$

The grid average net flux SW_{net} has been computed previously by the radiative code

We need to (1) conserve energy and (2) take into account the value of the surface temperature (emissivity) of the sub-surface i

$$LW^i_{net}(T^i_s) = \epsilon_i(LW_{dn} - \sigma(T^i_s)^4)$$

$$T^{i4}_s \approx T^4_s + 4T^3_s(T - T^i_s)$$

$$\mathrm{LW}_{net}(T_s) = \epsilon(LW_{dn} - \sigma(T^i_s)^4) - 4\sigma T^3_s \sum \omega_i\epsilon_i(T^i_s - T_s) \qquad \epsilon = \sum_i \omega_i\epsilon_i$$

$$4\sigma T^3_s \sum \omega_i\epsilon_i(T^i_s - T_s) = 0$$ Energy conservation

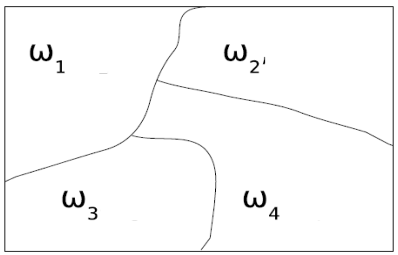
$$T_s = \frac{\sum \omega_i\epsilon_i T^i_s}{\epsilon}$$

Due to radiative code limitation, in LMDZ, we always must have $\varepsilon_i = 1$

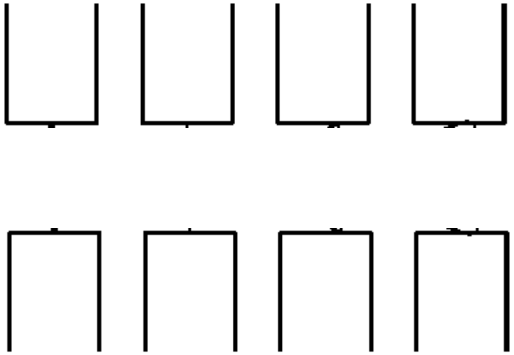
,

Each grid cell is divided into several sub-areas or ``sub-surfaces" of fractions

$$\sum_i \omega_i = 1$$



In the timestep :one PBL over **each** sub-surface



Vertical turbulent diffusion for moisture, heat , momentum

Time evolution of a variable X due to turbulent transport as the divergence of the turbulent flux:

$$\begin{cases} \frac{\partial X}{\partial t} & = -\frac{1}{\rho}\frac{\partial \Phi}{\partial z} \\ \Phi & = -\overline{w'X'} = -\rho k_z \frac{\partial X}{\partial z} \end{cases} \quad \text{with}$$

k_z : the vertical turbulent diffusion coefficient for X

in the atmosphere

$$\frac{\partial X}{\partial t} = -\frac{1}{\rho}\frac{\partial[-\rho k_z \frac{\partial X}{\partial z}]}{\partial z}$$

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Surface layer, constant flux: $u_*^2 = \frac{\tau}{\rho} = -\overline{u'w'},\ \theta^* = \frac{\overline{w'\theta'}}{u_*} = \frac{H}{\rho c_p u_*},\ \ q^* = \frac{\overline{w'q'}}{u_*}$

MOST, flux-gradient (Neutral) $\frac{\partial u}{\partial z}\frac{\kappa z}{u_*} = 1$ $\frac{\partial \Theta}{\partial z}\frac{\kappa z}{\Theta_*} = 1$

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MOST, flux-gradient (Neutral)

$$\frac{\partial u}{\partial z}\frac{\kappa z}{u_*} = 1 \qquad \frac{\partial \Theta}{\partial z}\frac{\kappa z}{\Theta_*} = 1$$

$$\int_{u_o}^{u_1} \partial u = \int_{z_o}^{z_1} \frac{u_*}{\kappa z}\,\mathrm{d}z \qquad \int_{\Theta_s}^{\Theta_1} \partial \Theta = \int_{z_o}^{z_1} \frac{\kappa \Theta_*}{z}\,\mathrm{d}z$$

$$u_1 - u_0 = \frac{u_*}{\kappa}\ln\frac{z_1}{z_0} \qquad \Theta_* = (\Theta_1 - \Theta_s)\frac{\kappa}{\ln\frac{z_1}{z_{0_\Theta}}}$$

$$H = \rho C_p \kappa^2 \frac{u_1}{\ln\frac{z_1}{z_{0_\Theta}}\ln\frac{z_1}{z_{0_u}}}(\Theta_1 - \Theta_s)$$

Vertical turbulent diffusion for moisture, heat , momentum

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k_z : the vertical turbulent diffusion coefficient for X

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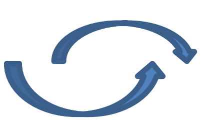
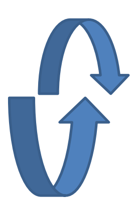
$$\int_{u_o}^{u_1} \partial u = \int_{z_o}^{z_1} \frac{u_*}{\kappa z}\,\mathrm{d}z \qquad \int_{\Theta_s}^{\Theta_1} \partial \Theta = \int_{z_o}^{z_1} \frac{\kappa \Theta_*}{z}\,\mathrm{d}z$$

$$u_1 - u_0 = \frac{u_*}{\kappa}\ln\frac{z_1}{z_0} \qquad \Theta_* = (\Theta_1 - \Theta_s)\frac{\kappa}{\ln\frac{z_1}{z_{0_\Theta}}}$$



stable



instable

$$H = \rho C_p \kappa^2 \frac{u_1}{\ln\frac{z_1}{z_{0_\Theta}}\ln\frac{z_1}{z_{0_u}}} F_{stab(R_i,z_0)}(\Theta_1 - \Theta_s)$$

Numerical world

Vertical discretization :

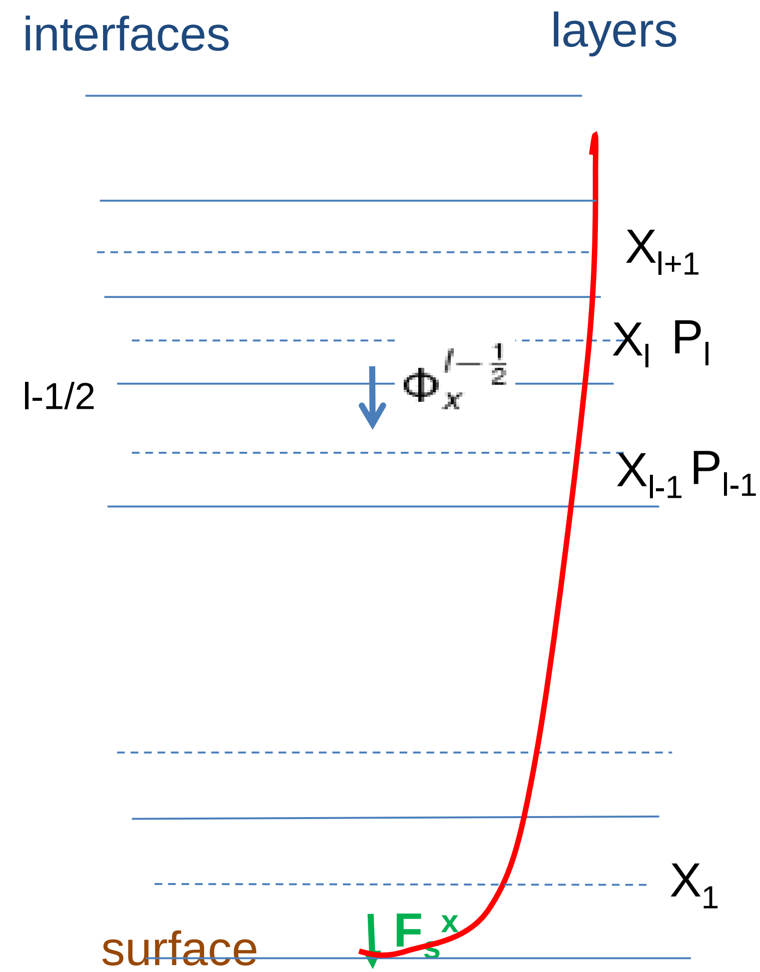
$$\Phi_x^{l-\frac{1}{2}} = -\rho_{l-\frac{1}{2}} k_{l-\frac{1}{2}} \frac{(X_l - X_{l-1})}{z_l - z_{l-1}} = -K_{l-\frac{1}{2}}(X_l - X_{l-1})$$

\+ Time discretization

$$\frac{X_l^{t+\delta t} - X_l^t}{\delta t} = -\frac{g}{\delta P_l}\left(\Phi_x^{l+\frac{1}{2},t+\delta t} - \Phi_x^{l-\frac{1}{2},t+\delta t}\right)$$

$$\begin{cases} -K_{l-\frac{1}{2}} X_{l-1}^{t+\delta t} + \left(\frac{m}{\delta t} + K_{l+\frac{1}{2}} + K_{l-\frac{1}{2}}\right) X_l^{t+\delta t} + K_{l+\frac{1}{2}} X_{l+1}^{t+\delta t} = \frac{m_l}{\delta t} X_l^t \\ F_s^{x,t+\delta t} = \rho |\vec{v}| C_d \left(X_1^{t+\delta t} - X_s^{t+\delta t}\right) \end{cases}$$

Flux >0 downward

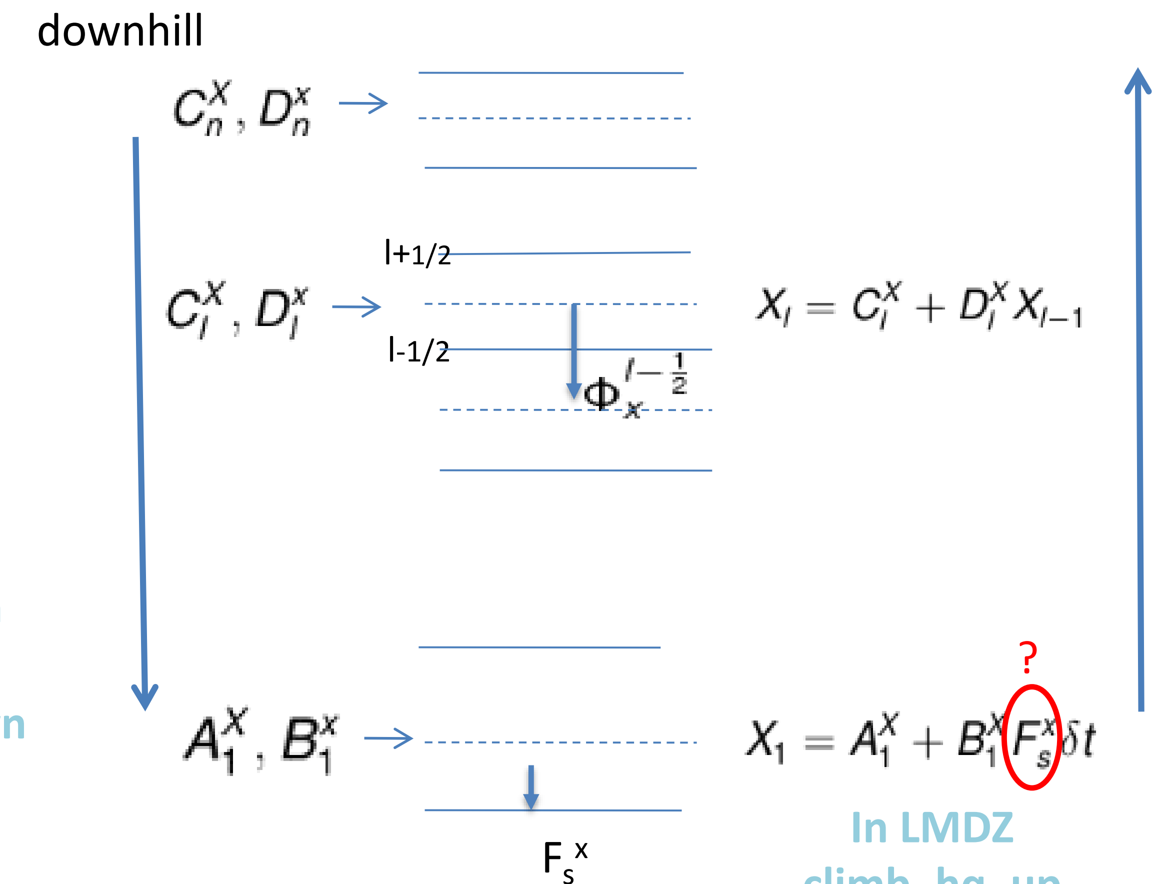


Tri-diagonal system with implicit boundary condition that can be solved for the vector
X= Enthalpy, specific humidity, wind

$\delta P = (P_{l-1} - P_l) = \rho g \delta z = m_l\, g$

$\Phi_X^{l-1/2} = -K_{l-1/2}(X_l - X_{l-1})$

$C_l^X, D_l^X, A_1^X, B_1^X$ depend only on properties in the layers above and the variables at the previous time step.



In LMDZ
climb_hq_down
(calc_coef)
climb_wind_down

In LMDZ
climb_hq_up
climb_wind_up

X= wind, enthalpie, specific humidity, tracers

Once F_1^x (flux of water mass, heat between the surface and the atmosphere) is known (boundary limit) , the X_i can be computed from the first layer to the top of the PBL

Sensible heat flux

$$H = C_p\Theta = C_p T\left(\frac{P}{P_o}\right)^{\kappa}$$

$$\begin{cases} F_s^{H,t+\delta t} = \rho|\vec{v}|C_d(H_1^{t+\delta t} - H_s^{t+\delta t}) \\ H_1^{t+\delta t} = A_1^{H,t} + B_1^{H,t}F_s^{H,t+\delta t}\delta t \end{cases}$$

$$F_s^{H,t+\delta t} = \frac{A_1^{H,t}}{\frac{1}{\rho|\vec{v}|C_d} - B_1^{H,t}\delta t} - \frac{\rho|\vec{v}|C_d}{\frac{1}{\rho|\vec{v}|C_d} - B_1^{H,t}\delta t}H_s^{t+\delta t}$$

$$F_s^{H,t+\delta t} = M_H^t + N_H^t T_s^{t+\delta t}$$

$$C_d = \kappa^2 / \left(\ln\left(\frac{z}{z0m}\right) * \ln\left(\frac{z}{z0m}\right)\right) * Fstab$$

Latent heat flux

$$\begin{cases} L_s^{q,t+\delta t} = \rho|\vec{v}|\beta C_d(q_1^{t+\delta t} - q_{sat}(T_s^{t+\delta t})) \\ q_1^{t+\delta t} = A_1^{q,t} + B_1^{q,t}L_s^{q,t+\delta t}\delta t \end{cases}$$

$$q_{sat}(T_s^{t+\delta t}) = q_{sat}(T_s) + \frac{\partial q_{sat}}{\partial T}\Big|_{T_s^t}(T_s^{t+\delta t} - T_s^t)$$

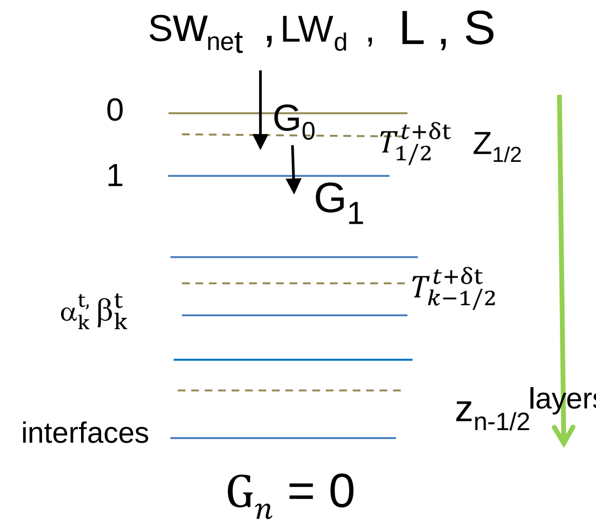
$$L_s^{q,t+\delta t} = M_q^t + N_q^t T_s^{t+\delta t}$$

C_d^x drag coefficient (Monin Obukhov, constant flux in the surface layer) depends on

- roughness lenghts (gustiness, vegetation)
- Richardson number (boundary layer stability)
- Formulation depends on the sub-surface type

Heat conduction (1) $$\begin{cases} \dfrac{\partial T}{\partial t} = -\dfrac{1}{c_v}\dfrac{\partial G}{\partial z} \\ G = -\lambda \dfrac{\partial T}{\partial z} \end{cases}$$

- Discretization of (1) in space and time
- $T^{t+\delta t}_{k+1/2} = \alpha^t_k \, T^{t+\delta t}_{k-1/2} + \beta^t_k$
- (1) for the first interface $G_0 = f(\lambda_i, T^{t+\delta t}_{i+1/2}, \delta z_i, \delta z_{i+1/2})$



Surface energy budget

$LW_{net} + SW_{net} + F_{sens} + F_{lat} = G_0$

Heat conduction

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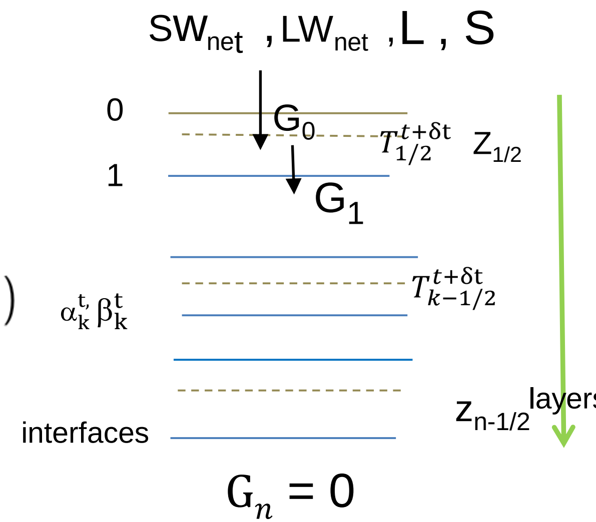

Surface energy budget

$$LW_{net} + SW_{net} + F_{sens} + F_{lat} + G_0 = 0$$

$$C^t_{p_{1/2}} \frac{T^{t+\delta t}_{1/2} - T^t_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{T^{t+\delta t}_{3/2} - T^{t+\delta t}_{1/2}}{z_{3/2} - z_{1/2}}\right] + \sum F^{\downarrow}\left(T^{t+\delta t}_S\right) + Rad - \varepsilon\sigma\left(T^{t+\delta t}_S\right)^4$$

$$F^{H,t+\delta t}_s = M^t_H + N^t_H T^{t+\delta t}_s \qquad L^{q,t+\delta t}_s = M^t_q + N^t_q T^{t+\delta t}_s$$

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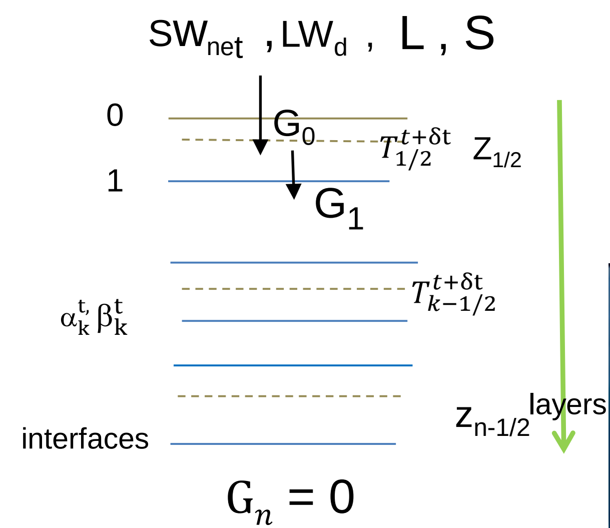

Surface energy budget

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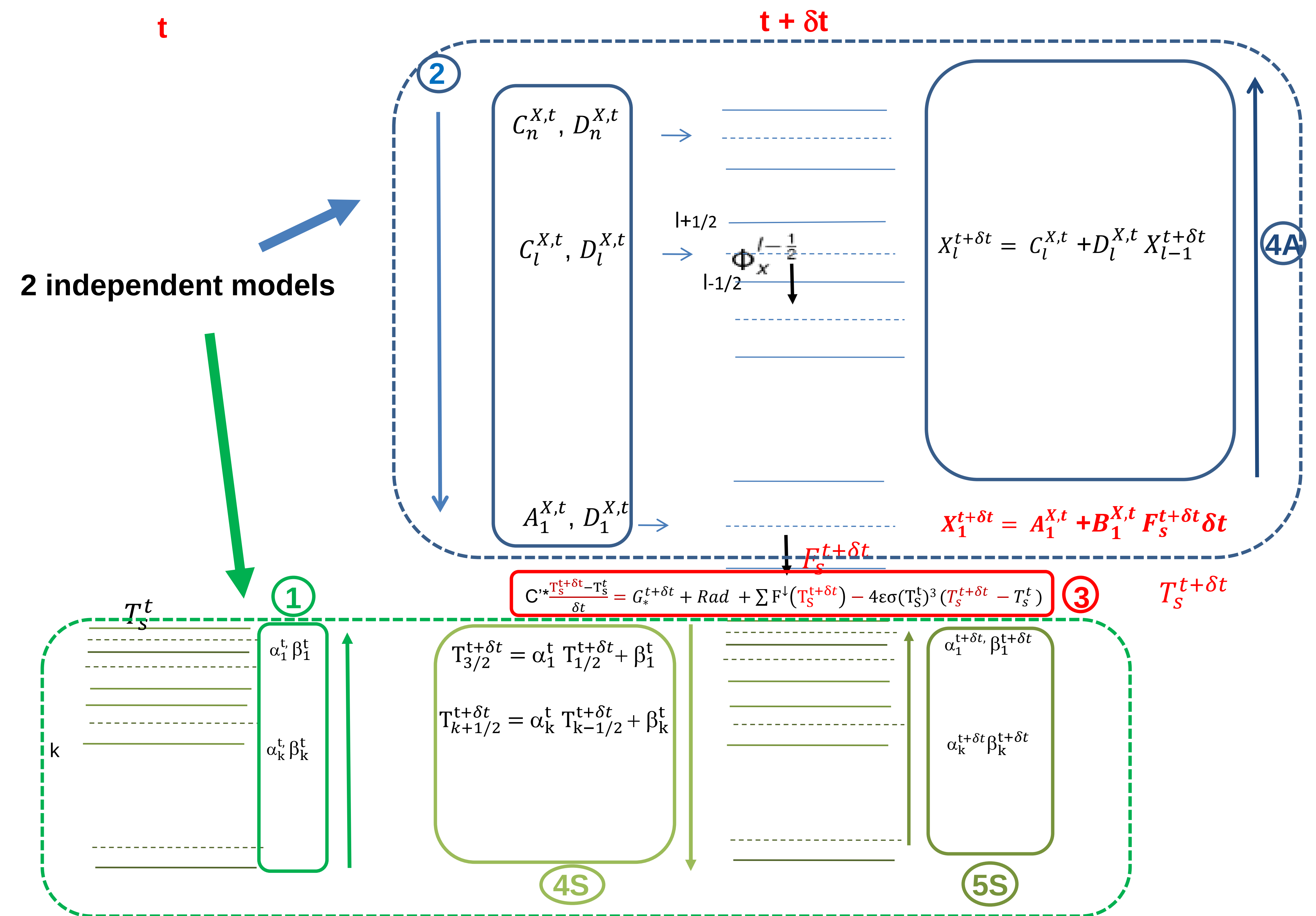
$$C^t_{p_{1/2}} \frac{T^{t+\delta t}_{1/2} - T^t_{1/2}}{\delta t} = \frac{1}{z_1 - z_0}\left[\lambda_1 \frac{T^{t+\delta t}_{3/2} - T^{t+\delta t}_{1/2}}{z_{3/2} - z_{1/2}}\right] + \sum F^{\downarrow}\left(T^{t+\delta t}_S\right) + Rad - \varepsilon\sigma\left(T^{t+\delta t}_S\right)^4$$

$$F^{H,t+\delta t}_s = M^t_H + N^t_H T^{t+\delta t}_s \qquad L^{q,t+\delta t}_s = M^t_q + N^t_q T^{t+\delta t}_s$$

Temperature continuity : T_s extrapolated from soil temperature

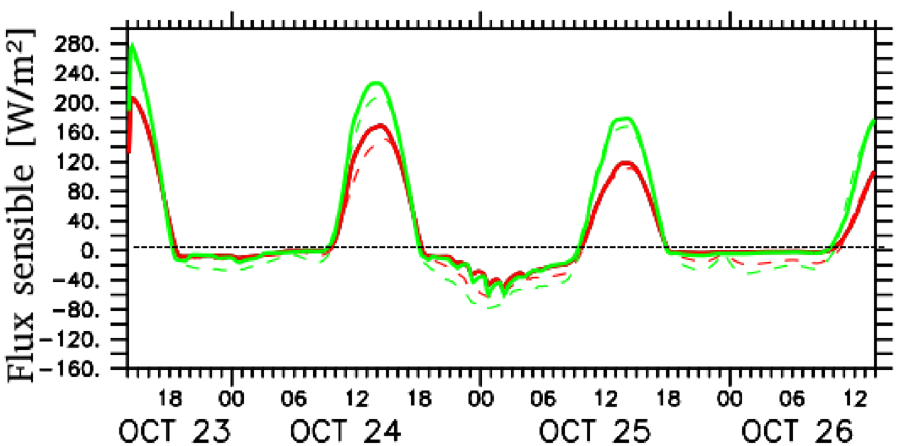
$$C'^{*}\frac{T^{t+\delta t}_S - T^t_S}{\delta t} = G^{t+\delta t}_{*} + Rad + \sum F^{\downarrow}\left(T^{t+\delta t}_S\right) - 4\varepsilon\sigma\left(T^t_S\right)^3\left(T^{t+\delta t}_s - T^t_s\right)$$

One diffusion equation from the top of the PBL to the bottom of the soil

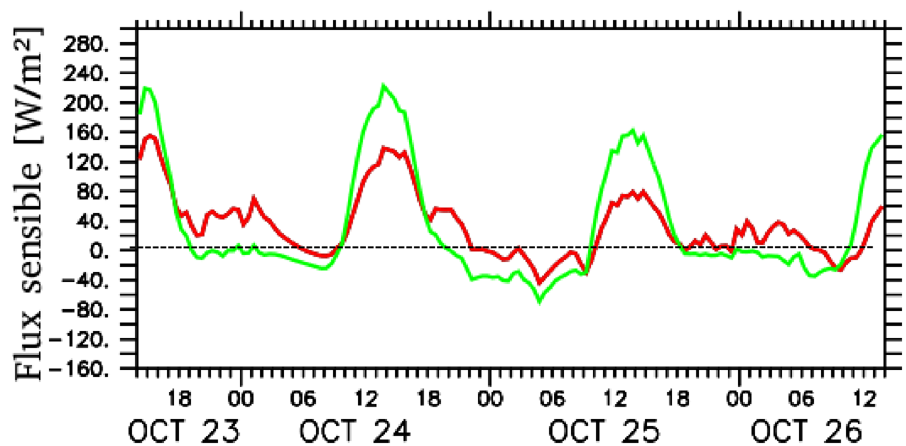


Some examples : 1D south Great Plaines

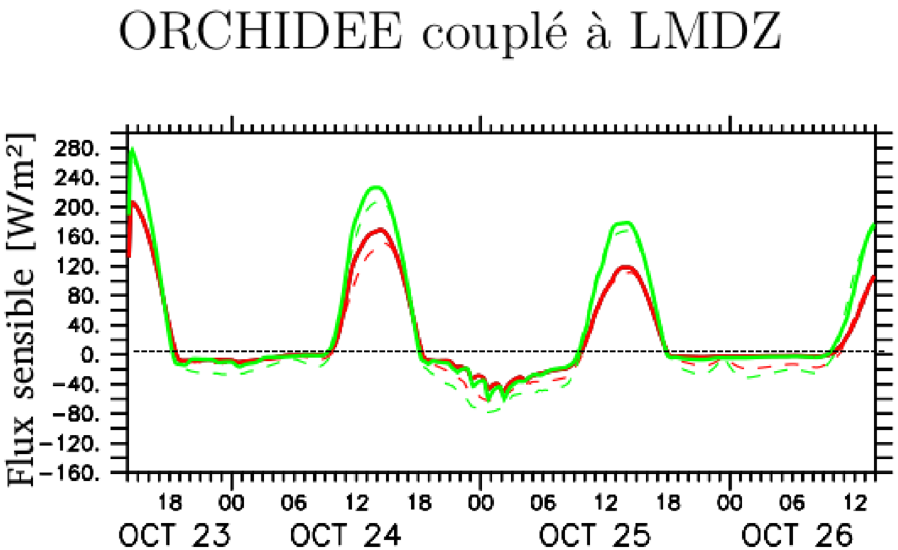
ORCHIDEE couplé à LMDZ

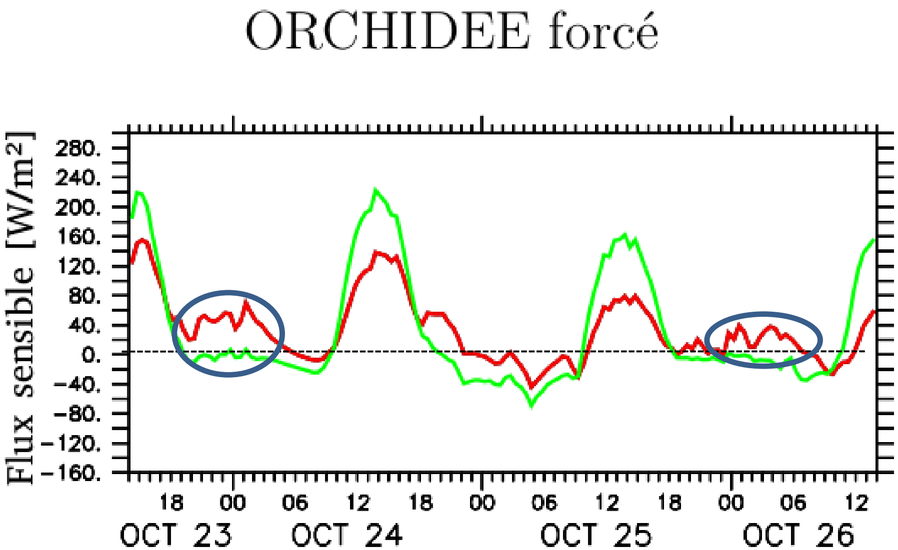


ORCHIDEE forcé



PhD S. Ait Mesbah





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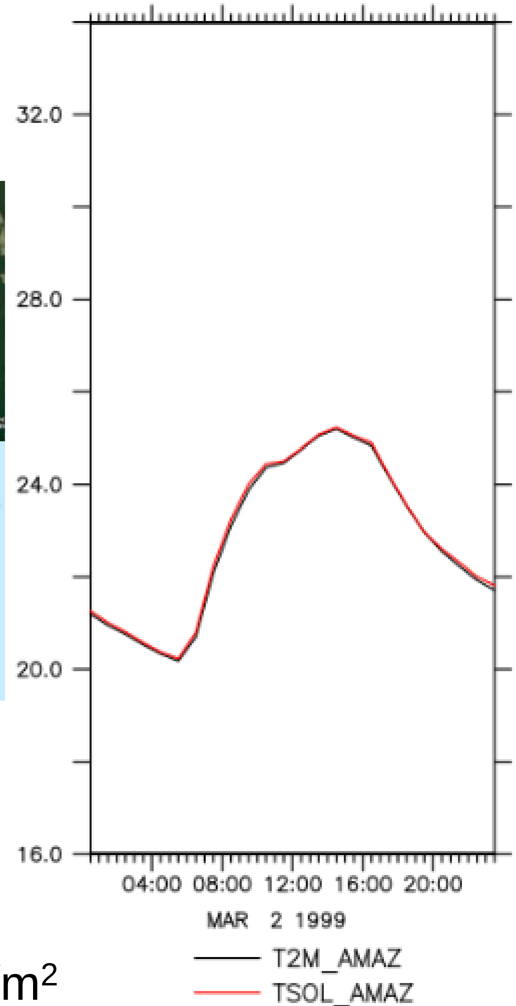
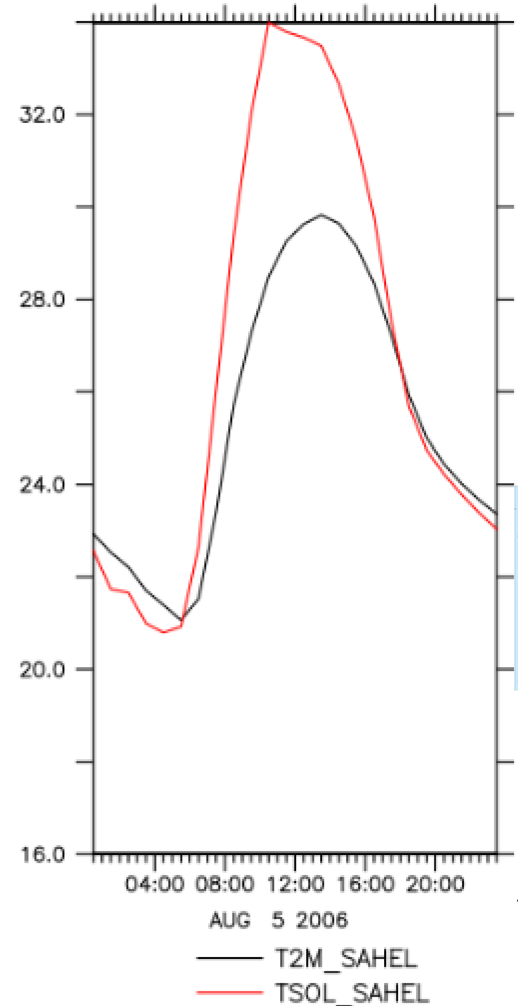
Same physics : different local conditions

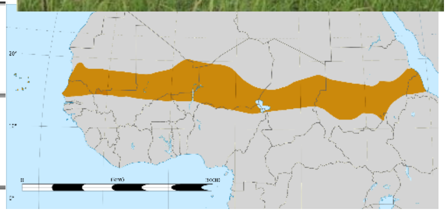
Amazonie



Albedo 0.12
Z_0 =1.8m
Soil moisture:50kg/m^2

Sahel

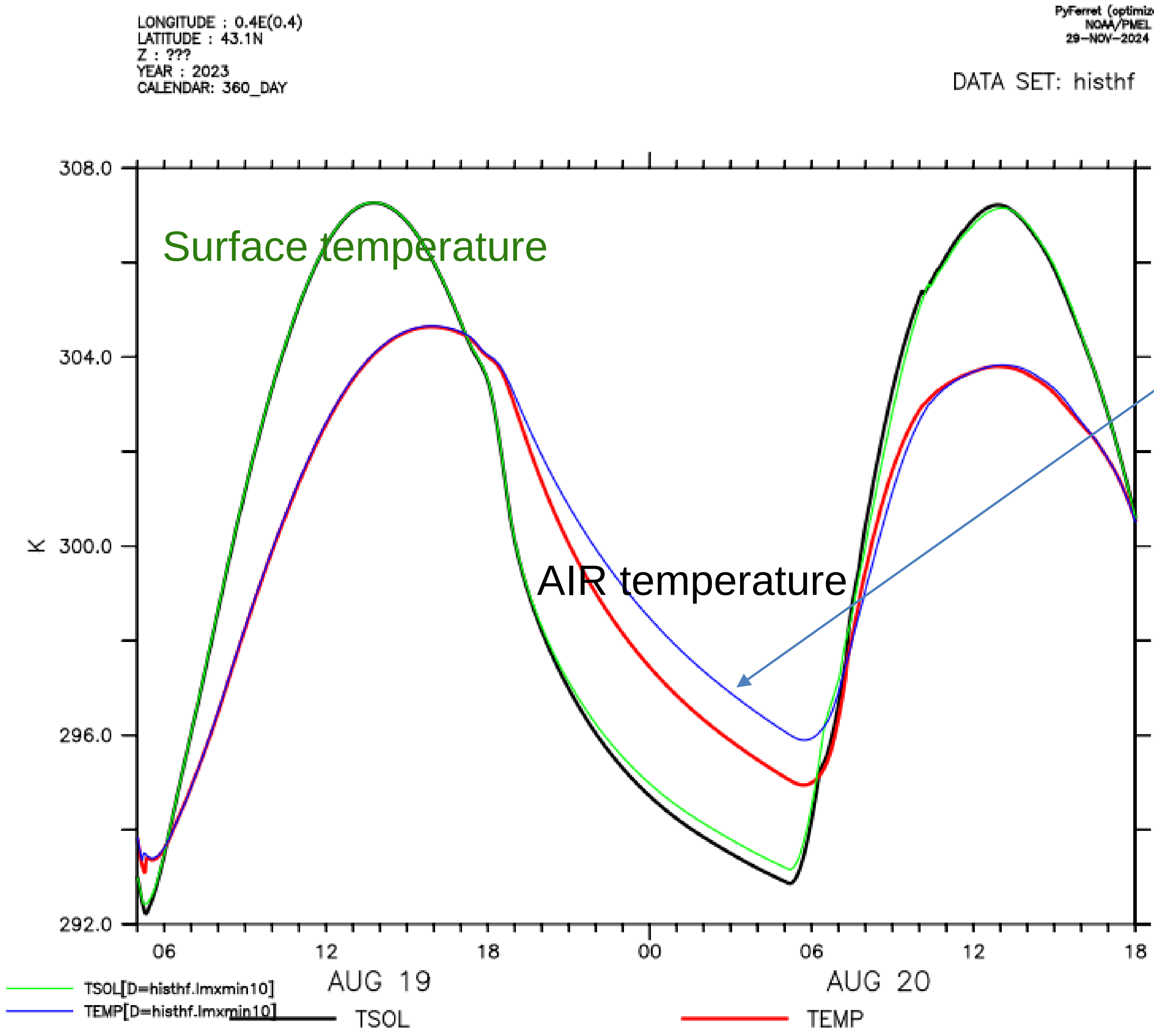




Albedo=0.26
Z_0 =0.01m
Soil moisture:25kg/m^2

Thanks :Rachida El Ouaraini and Bouchra Sadiki,UM6P 2022

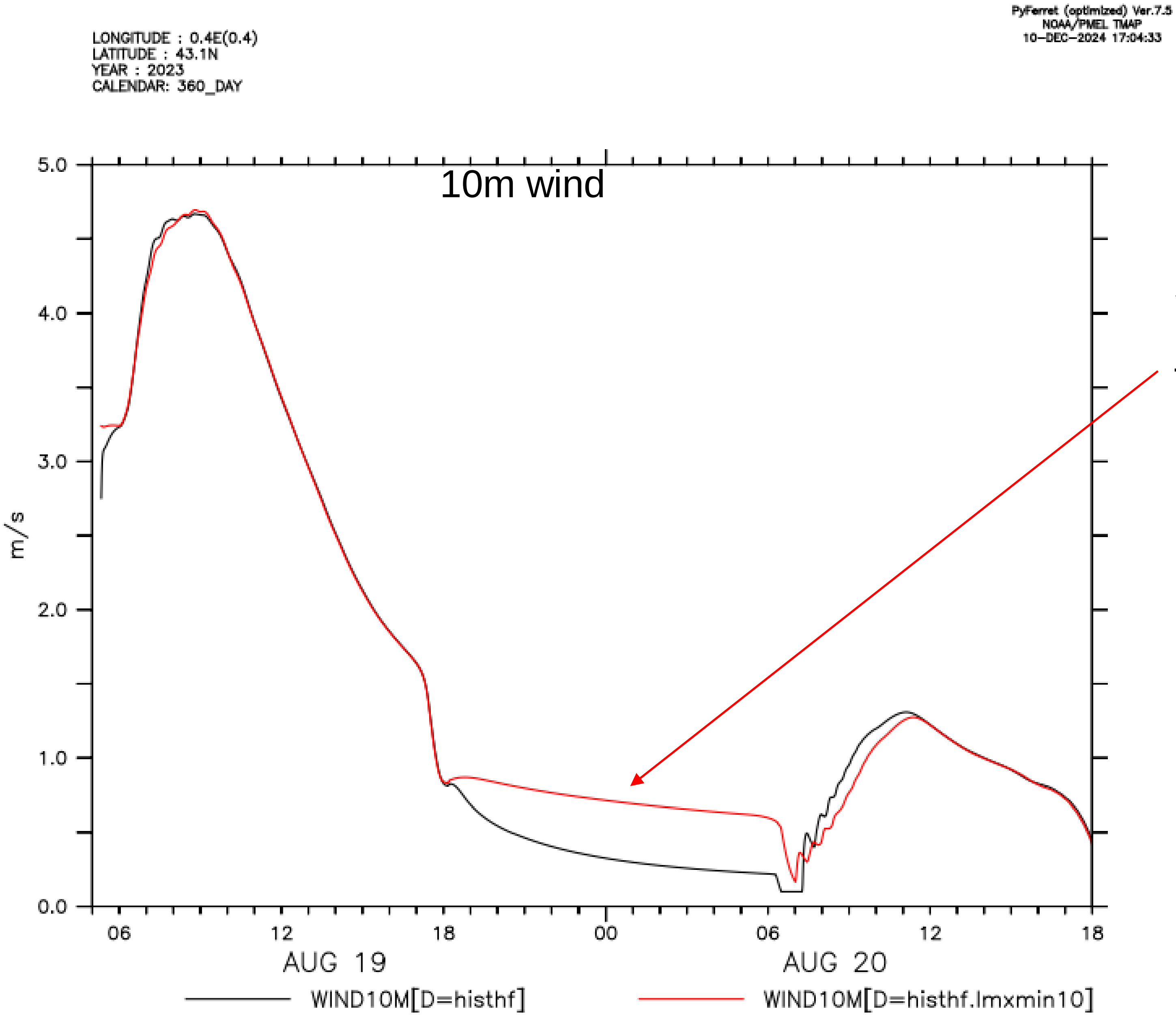
2 clear sky days in summer during MOSAI campaign in Lannemezan (Near Pyrénées)



Same physics authorizing less decoupling between The land surface and the atmosphere

1D Case, DEPHY group, A. Maison

2 clear sky days in summer during MOSAI campaign in Lannemezan (Near Pyrénées)



Same physics authorizing less decoupling between The land surface and the atmosphere

1D Case, DEPHY group, A. Maison

Atmosphere/surface coupling in LMDZOR

LMDZ (phylmd)

Planetary boundary layer and surface modules

pbl_surface

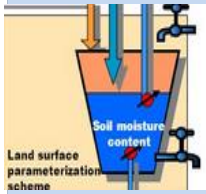
(A_q , $B_{q,}$ A_H , $B_{B,}$ $C_{dh,}$, A_u , $B_{u,}$ A_v , B_v , C_{dh}, T_1, $q_{1,}$ u_1,$v_{1,}$ $LW_{net,}$ $LW_{down,}$ Sw_{net})

AcoefH, AcoefQ, BcoefH, BcoefQ cdragh,lwdown,swnet

,

(is_ter, ok_veget = n)

surf_land_bucket

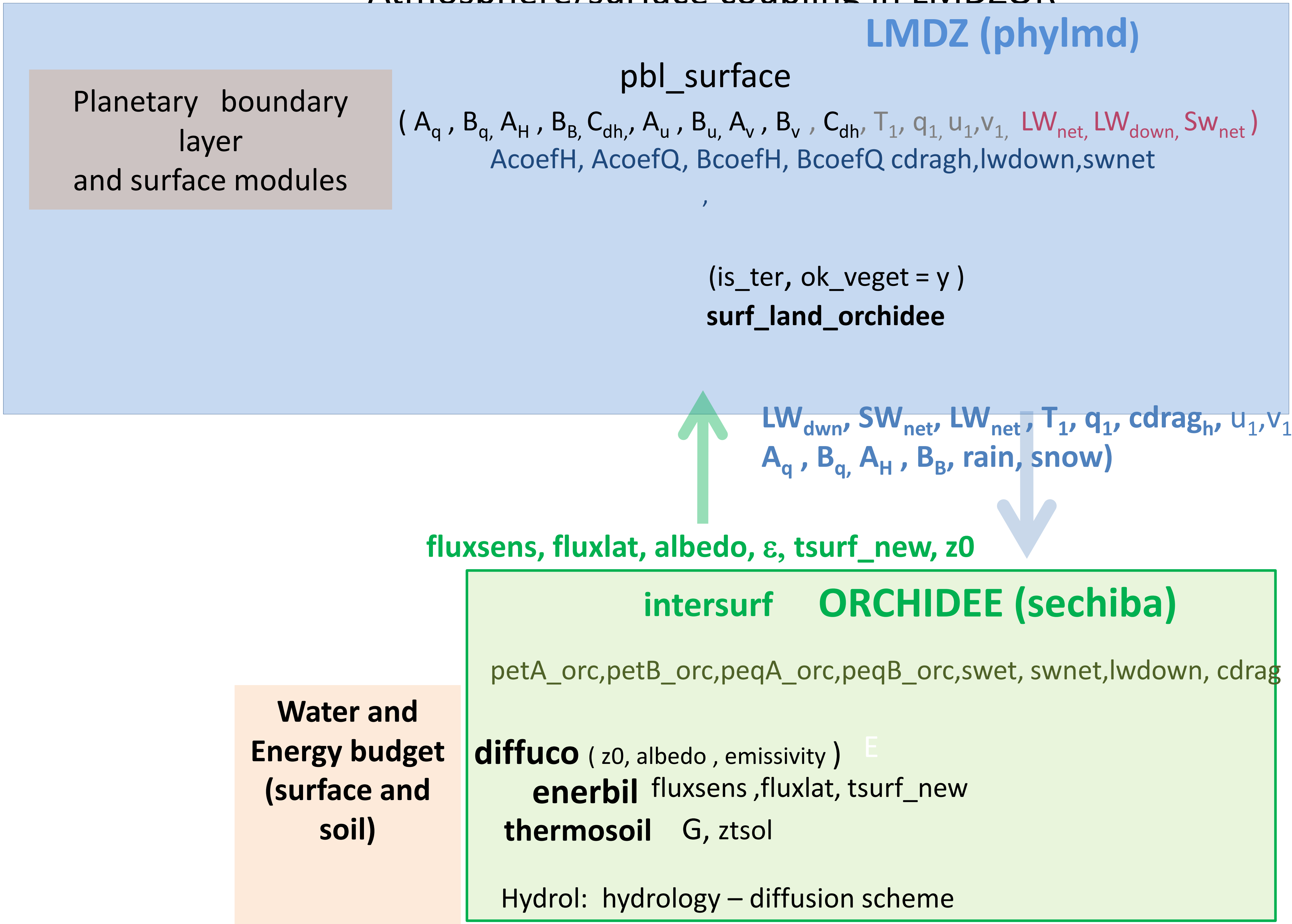


(soil.F90: soil T, heat capacity, conduction,
calcul_flux : sens,flat,tsurf_new
Hydro= water budget (snow, precip, Evap)

(is_ter, ok_veget = y)

surf_land_orchidee

Atmosphere/surface coupling in LMDZOR



Atmosphere/surface coupling in LMDZOR

LMDZ (phylmd)

Planetary boundary layer and surface modules

pbl_surface

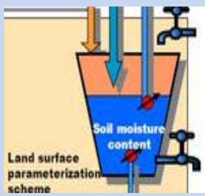
(A_q , $B_{q,}$ A_H , $B_{B,}$ $C_{dh,}$, A_u , $B_{u,}$ A_v , B_v , C_{dh}, T_1, q_1, u_1,v_1, LW_{net}, LW_{down}, Sw_{net})

AcoefH, AcoefQ, BcoefH, BcoefQ cdragh,lwdown,swnet

,

(is_ter, ok_veget = n)

surf_land_bucket



(soil.F90: soil T, heat capacity, conduction,

calcul_flux : sens,flat,tsurf_new

Hydro= water budget (snow, precip, Evap)

(is_ter, ok_veget = y)

surf_land_orchidee

LW_{dwn}, SW_{net}, LW_{net} , T_1, q_1, $cdrag_h$, u_1,v_1

A_q , $B_{q,}$ A_H , B_B, rain, snow)

fluxsens, fluxlat, albedo, ε, tsurf_new, z0

intersurf **ORCHIDEE (sechiba)**

petA_orc,petB_orc,peqA_orc,peqB_orc,swet, swnet,lwdown, cdrag

Water and Energy budget (surface and soil)

diffuco (z0, albedo , emissivity)

enerbil fluxsens ,fluxlat, tsurf_new

thermosoil G, ztsol

Hydrol: hydrology – diffusion scheme

Call tree

In subroutine PHYSIQ
loop over time steps
CALL change_srf_frac : Update fraction of the sub-surfaces (pctsrf)
....
CALL pbl_surface Main subroutine for the interface with surface
Calculate net radiation at sub-surface
Loop over the sub-surfaces nsrf
Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface in not zero)
CALL cdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)
CALL coef_diff_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)
CALL climb_hq_down downhill for enthalpy H and humidity Q
CALL climb_wind_down downhill for wind (U and V)
CALL surface models for the various surface types: **surf_land**, **surf_landice**, **surf_ocean** **or** **surf_seaice**.
Each surface model computes:
- evaporation, latent heat flux, sensible heat flux, momentum
- surface temperature, albedo (emissivity), roughness lengths

CALL climb_hq_up : compute new values of enthalpy H and humidity Q
CALL climb_wind_up : compute new values of wind (U and V)
Uncompress variables : (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)
Cumulate in global variables after weighting by sub-surface fractions
Surface diagnostics : (T, q, wind are evaluated at a reference level (2m) owing to an interpolation scheme based on the MO laws).
End Loop over the sub-surfaces
Calculate the mean values over all sub-surfaces for some variables
End pbl-surface

Take home messages

- Atmosphere and surfaces are coupled through turbulent diffusion and radiation
- Different sub-surface are considered (albedo, emissivity, rugosity) for ocean, land, land-ice, sea ice but only one atmosphere is above.
- For each sub-surface one solves a unique diffusion equation from the top of the PBL to the bottom of the soil with an implicit scheme. The surface energy budget allows to find the boundary conditions
- Priority is given to the energy conservation
- The coupling matters : surface forced simulations can produce irrealistic surface fluxes, local surface condition strongly impact near surface variables

- Technical note **:** Description of the interface with the surface and the computation of the turbulent diffusion in LMDZ (J.L.Dufresne and J. Ghattas) https://forge.ipsl.jussieu.fr/orchidee/attachment/wiki/Documentation/CouplingLMDZ/Dufresne,%20Ghattas%20-%202009_Coupling-ORC-LMDZ.pdf
- Thèse F. Hourdin 1993 (section 3.3.3 and annexes) web page F. Hourdin
- Wang F., F. Cheruy, J.L. Dufresne, 2016: The improvement of soil thermodynamics and its effects on land surface meteorology in the IPSL climate model. Geosci. Model Dev., 9, 363–381, 2016 www.geosci-model-dev.net/9/363/2016/
- Cheruy et al.,2020 Improved near surface continental climate in IPSL-CM6A-LR by combined evolutions of atmospheric and land surface physics https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2019MS002005

Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$\left(\delta P_l + R_{l+1}^X + R_l^X\right) X_l = \delta P_l \ X_l^0 + R_{l+1}^X \ X_{l+1} + R_l^X \ X_{l-1} \quad (2 \leq l < n)$$

with $R_l^X = g \delta t K_l$

At the top (l=n, Φ_n=0)

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n \ X_n^0 + R_n^X \ X_{n-1}$$

At the bottom: (l=1) : $m_1 \frac{X_1 - X^0_1}{\delta t} = \Phi^1_x - \Phi^2_x$

$$m_1 \frac{X_1 - X_1^0}{\delta t} = K_2(X_2 - X_1) - F_1^X$$

$$\left(\delta P_1 + R_1^X\right) X_1 = \delta P_1 \ X_1^0 + R_2^X \ X_2 - g \delta t F_1^X$$

With F_1^X : flux of X at the bottom of the first layer (i.e. between the surface and the atmosphere), positive downward.

$$K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l}$$

Solving the tridiagonal system

Starting from top:

$$\left(\delta P_n + R_n^X\right) X_n = \delta P_n \; X_n^0 + R_n^X \; X_{n-1}$$

can be written as:

$$\boxed{X_n = C_n^X + D_n^X X_{n-1}}$$

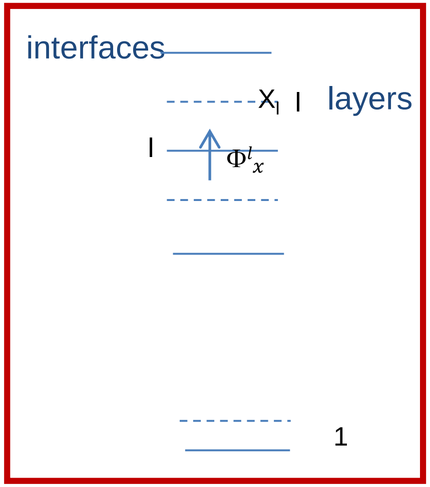
with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$

with $R_l^X = g\delta t K_l$

$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$

Solving the tridiagonal system



$$X_{l+1} = C^X_{l+1} + D^X_{l+1} X_l$$

$$\left(\delta P_l + R^X_{l+1} + R^X_l\right) X_l \;=\; \delta P_l \; X^0_l + R^X_{l+1} \; X_{l+1} + R^X_l \; X_{l-1} \quad (2 \leq l < n$$

$$\left(\delta P_l + R^X_{l+1}\left(1 - D^X_{l+1}\right) + R^X_l\right) X_l \;=\; \delta P_l \; X^0_l + R^X_{l+1} \; C^X_{l+1} + R^X_l \; X_{l-1}$$

with $R^X_l = g\delta t K_l$

So we obtain by reccurence:

$$X_l = C^X_l + D^X_l X_{l-1} \qquad (2 \leq l \leq n)$$

with, for $(2 \leq l < n)$

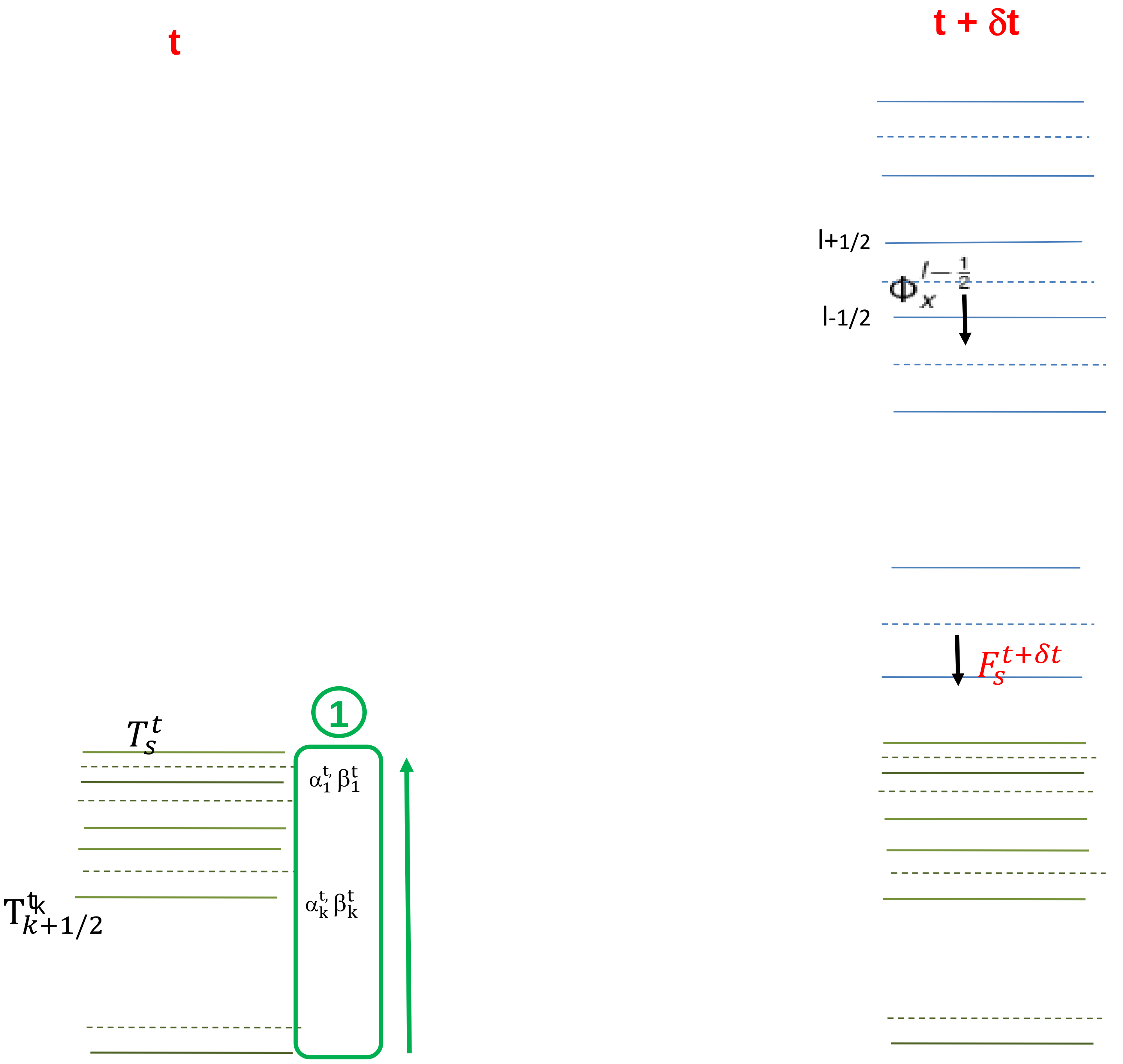
$$\begin{cases} C^X_l = \dfrac{X^0_l \delta P_l + R^X_{l+1} C^X_{l+1}}{\delta P_l + R^X_l + R^X_{l+1}(1 - D^X_{l+1})} \\[2ex] D^X_l = \dfrac{R^X_l}{\delta P_l + R^X_l + R^X_{l+1}(1 - D^X_{l+1})} \end{cases}$$

depend only on properties in the layers above and the variables at the previous time step.

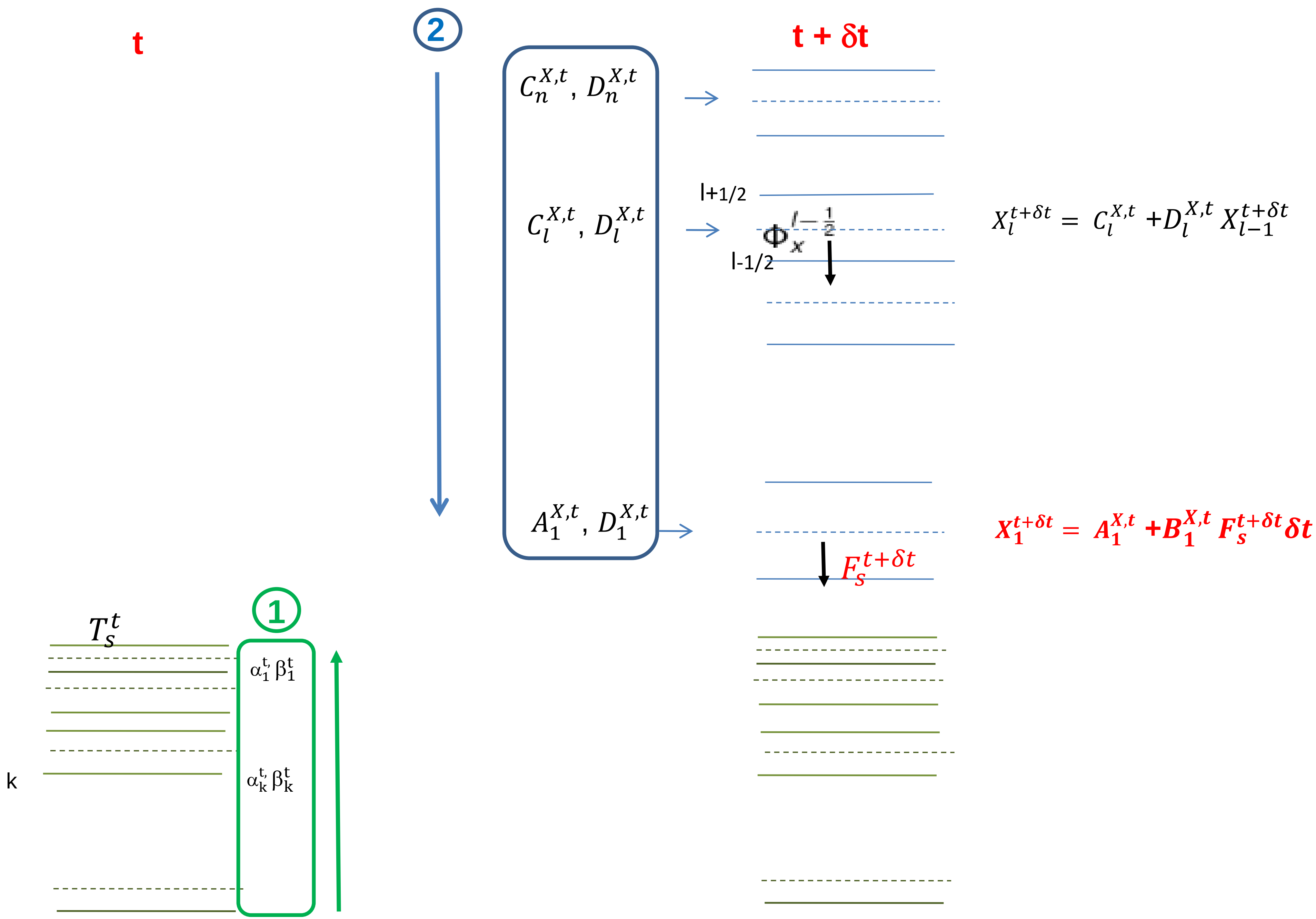
In LMDZ routine calc_coe

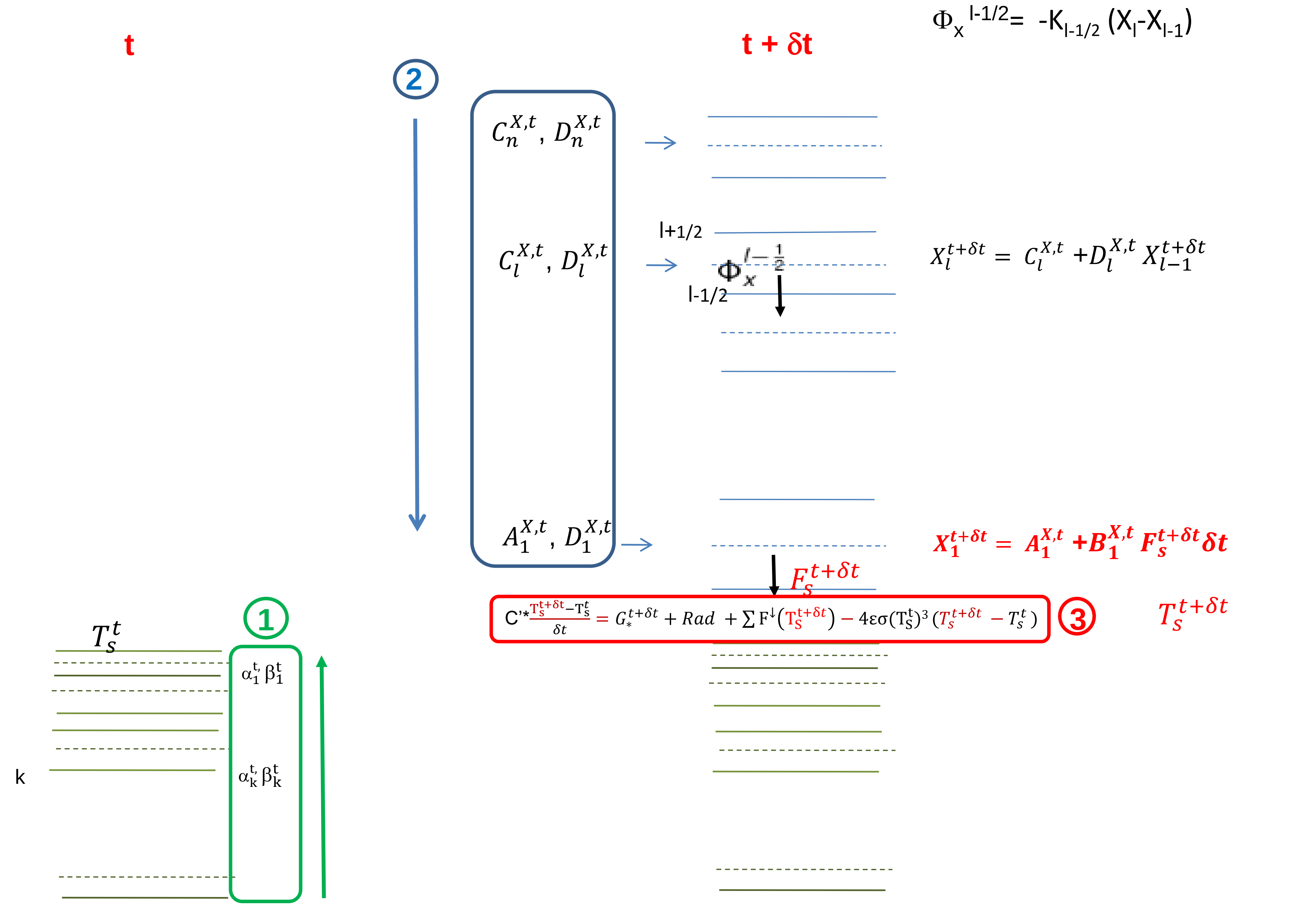
Fully coupled system : Recap

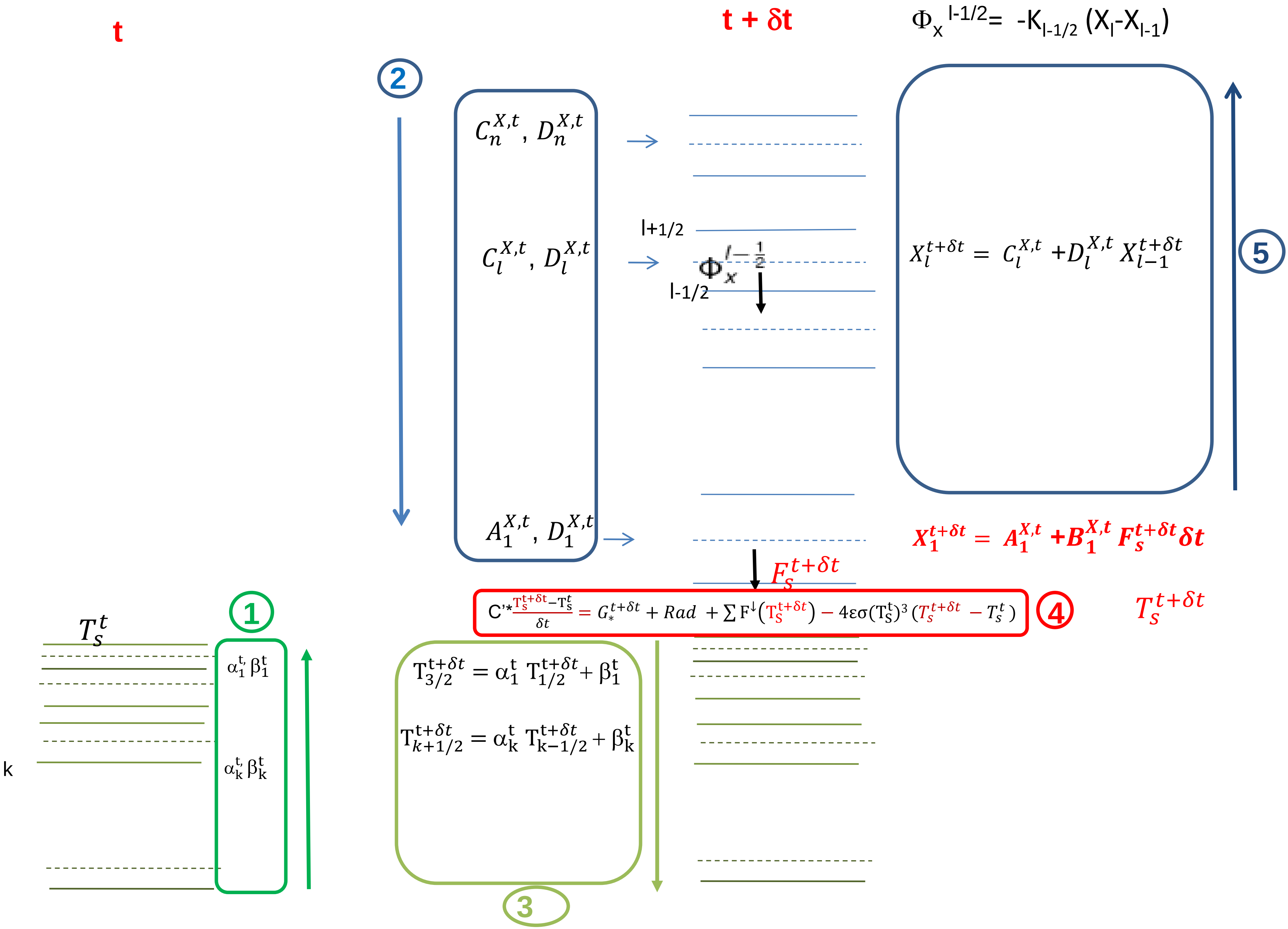
$$\Phi_x^{l-1/2} = -K_{l-1/2}\,(X_l - X_{l-1})$$



Fully coupled system : Recap




$\Phi_X^{l-1/2} = -K_{l-1/2}(X_l - X_{l-1})$
t
t + δt
2
$C_n^{X,t}, D_n^{X,t}$
$C_l^{X,t}, D_l^{X,t}$
l+1/2
$\Phi_x^{l-\frac{1}{2}}$
l-1/2
$X_l^{t+\delta t} = C_l^{X,t} + D_l^{X,t} X_{l-1}^{t+\delta t}$
$A_1^{X,t}, D_1^{X,t}$
$X_1^{t+\delta t} = A_1^{X,t} + B_1^{X,t} F_s^{t+\delta t} \delta t$
$F_s^{t+\delta t}$
$C'* \frac{T_S^{t+\delta t} - T_S^t}{\delta t} = G_*^{t+\delta t} + Rad + \sum F^{\downarrow}(T_S^{t+\delta t}) - 4\varepsilon\sigma(T_S^t)^3 (T_s^{t+\delta t} - T_s^t)$
3
$T_s^{t+\delta t}$
T_s^t
1
α_1^t, β_1^t
k
α_k^t, β_k^t


t
t + δt
$\Phi_X^{l-1/2} = -K_{l-1/2}(X_l - X_{l-1})$
2
$C_n^{X,t}, D_n^{X,t}$
$C_l^{X,t}, D_l^{X,t}$
$A_1^{X,t}, D_1^{X,t}$
l+1/2
$\Phi_x^{l-\frac{1}{2}}$
l-1/2
$X_l^{t+\delta t} = C_l^{X,t} + D_l^{X,t} X_{l-1}^{t+\delta t}$
5
$X_1^{t+\delta t} = A_1^{X,t} + B_1^{X,t} F_s^{t+\delta t} \delta t$
$F_s^{t+\delta t}$
$C'^* \frac{T_S^{t+\delta t} - T_S^t}{\delta t} = G_*^{t+\delta t} + Rad + \sum F^{\downarrow}(T_S^{t+\delta t}) - 4\varepsilon\sigma (T_S^t)^3 (T_S^{t+\delta t} - T_S^t)$
4
$T_S^{t+\delta t}$
T_S^t
1
α_1^t, β_1^t
α_k^t, β_k^t
k
$T_{3/2}^{t+\delta t} = \alpha_1^t T_{1/2}^{t+\delta t} + \beta_1^t$
$T_{k+1/2}^{t+\delta t} = \alpha_k^t T_{k-1/2}^{t+\delta t} + \beta_k^t$
3

Fully coupled system : Recap