

# Atmosphere-surface coupling in LMDZ

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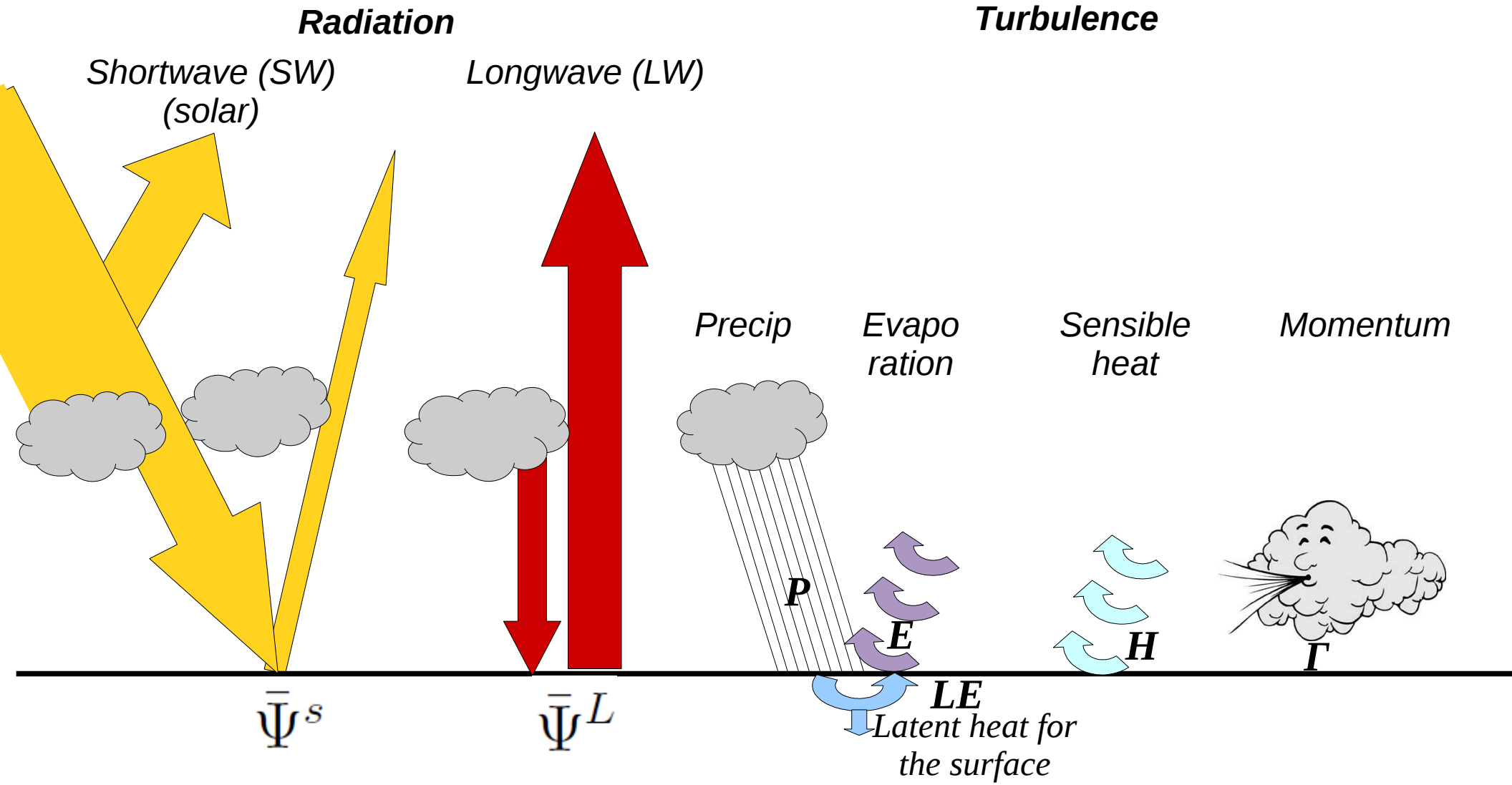
**Technical note** : Description of the interface with the surface and  
the computation of the turbulent diffusion in LMDZ  
[http://www.lmd.jussieu.fr/~jldufres/publi/pbl\\_surface.pdf](http://www.lmd.jussieu.fr/~jldufres/publi/pbl_surface.pdf)

*LMDZ training, 6 December 2016*

# Atmosphere-surface interactions

The **atmosphere and the surface are coupled** through *turbulence* (in boundary layer) and *radiation* (SW and LW). Currently, there are no direct influence of the surface to other parametrizations.

The surface “receive” precipitation from the atmosphere (no direct feedback).



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The surface “receive” precipitation from the atmosphere (no direct feedback).

The surface impacts the atmosphere via the orography (factors constant with time)

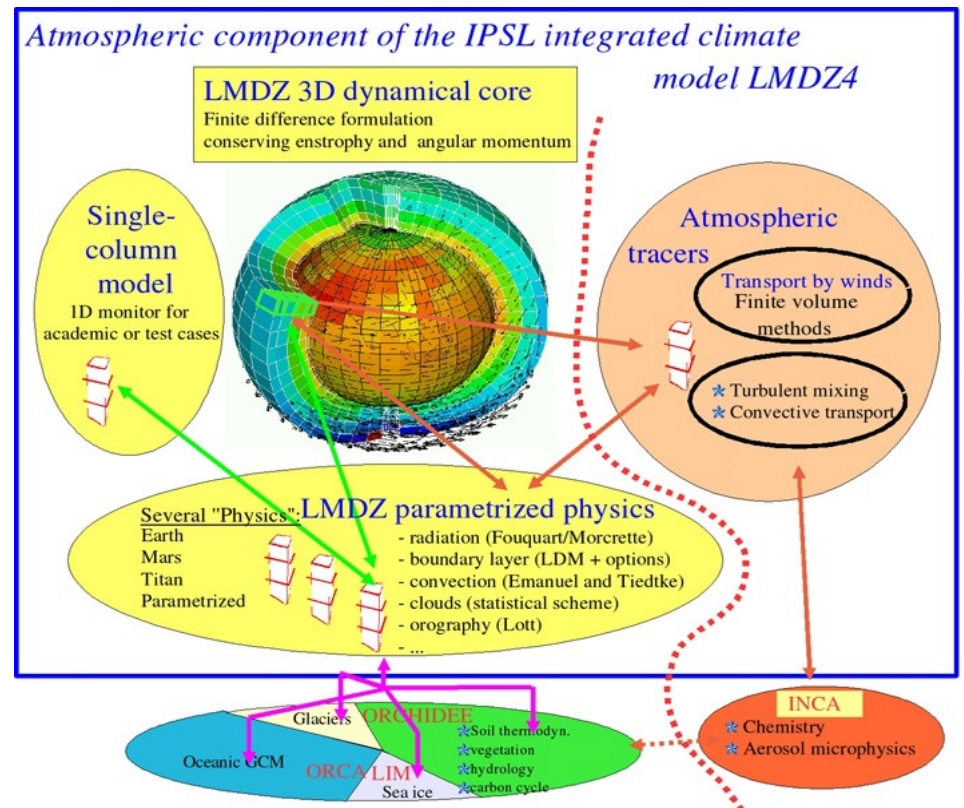
## In LMDZ:

Each surface grid may be decomposed in a maximum of 4 sub-grids of different types: land (\_ter), continental ice (\_lic), open ocean (\_oce), sea-ice (\_sic)

*Radiation* depends only on mean surface properties

*Turbulent diffusion* depends on local sub-grid property

No influence of sub-surface properties to any other parameterization.



# Turbulent diffusion

The change of a variable  $X$  with time is:

$$\rho \partial_t X = -\partial_z \phi \quad (8)$$

where  $\rho$  is the volumic mass ( $kg.m^{-3}$ ) and  $\phi$  the flux of  $X$ . Variable  $X$  can be the specific humidity, moist static energy, momentum, tracer.... For the vertical diffusion, the flux  $\phi$  of  $X$  is defined as:

$$\phi = -\rho k_z \partial_z X \quad (9)$$

with  $k_z$  is the diffusion coefficient ( $m^2.s^{-1}$ ).

**Space discretization.** We consider  $n$  layers from  $l = 1$  (surface) to  $l = n$  (top of atmosphere, TOA) and  $n + 1$  interfaces from  $l = 1$  (surface) to  $l = n + 1$  (TOA). The space discretization of above equations gives

$$m_l \partial_t X_l = \phi_l - \phi_{l+1} \quad (10)$$

$$\phi_l = -K_l (X_l - X_{l-1}) \quad (11)$$

with  $X_l$  average value of  $X$  for layer  $l$ ,  $m_l$  mass per unit surface ( $kg.m^{-2}$ ) of layer  $l$ ,  $\phi_l$  flux at interface  $l$  and

$$K_l = \frac{k_z \rho^2 g}{P_{l-1} - P_l} \quad (12)$$

with  $P_l$  pressure of layer  $l$ .

# Turbulent diffus

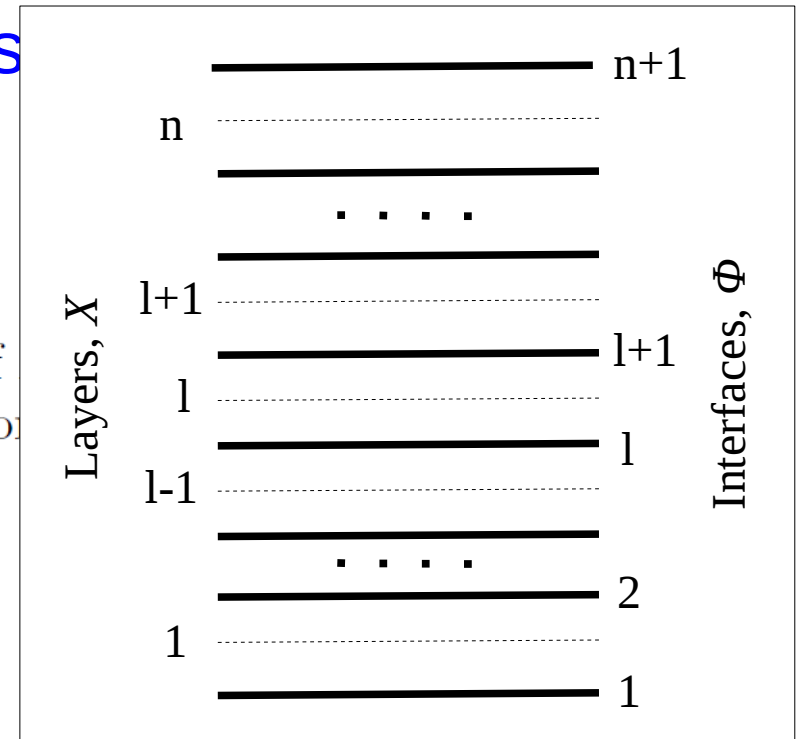
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## Turbulent diffusion

$$\begin{aligned} m_l \partial_t X_l &= \phi_l - \phi_{l+1} \\ \phi_l &= -K_l (X_l - X_{l-1}) \end{aligned}$$

**Time discretization.** We use an implicit scheme:

$$m_l \frac{X_l(t + \delta t) - X_l(t)}{\delta t} = \phi_l(t + \delta t) - \phi_{l+1}(t + \delta t)$$

$$\begin{aligned} m_l \frac{X_l - X_l^0}{\delta t} &= \phi_l - \phi_{l+1} \\ &= K_{l+1}(X_{l+1} - X_l) - K_l(X_l - X_{l-1}) \end{aligned}$$

with :  $X_l = X_l(t + \delta t)$  and  $X_l^0 = X_l(t)$ .

$$\left( \frac{m_l}{\delta t} + K_{l+1} + K_l \right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

Tridiagonal system that can be solved for vector  $X$

$$-K_l X_{l-1} + \left( \frac{m_l}{\delta t} + K_{l+1} + K_l \right) X_l + K_{l+1} X_{l+1} = \frac{m_l}{\delta t} X_l^0$$

# Solving the tridiagonal system

$$\left(\frac{m_l}{\delta t} + K_{l+1} + K_l\right) X_l = \frac{m_l}{\delta t} X_l^0 + K_{l+1} X_{l+1} + K_l X_{l-1}$$

which may be written as:

$$\boxed{(\delta P_l + R_{l+1}^X + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1}} \quad \underline{(2 \leq l < n)}$$

$$\text{with } R_l^X = g\delta t K_l$$

At the top

$$\boxed{(\delta P_n + R_n^X) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}}$$

At the bottom:

$$m_1 \frac{X_1 - X_1^0}{\delta t} = K_2(X_2 - X_1) - F_1^X$$

$$\boxed{(\delta P_1 + R_1^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t \underline{F_1^X}}$$

With  $F_1^X$  : flux of  $X$  at the bottom of the first layer (i.e. between the surface and the atmosphere), positive downward.

## Solving the tridiagonal system

Starting from top:

$$(\delta P_n + R_n^X) X_n = \delta P_n X_n^0 + R_n^X X_{n-1}$$

can be written as:

$$X_n = C_n^X + D_n^X X_{n-1}$$

with

$$C_n^X = \frac{X_n^0 \delta P_n}{\delta P_n + R_n^X}$$

$$D_n^X = \frac{R_n^X}{\delta P_n + R_n^X}$$



## Solving the tridiagonal system

$$X_{l+1} = C_{l+1}^X + D_{l+1}^X X_l$$

$$(\delta P_l + R_{l+1}^X + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X X_{l+1} + R_l^X X_{l-1} \quad (2 \leq l < n)$$

$$(\delta P_l + R_{l+1}^X (1 - D_{l+1}^X) + R_l^X) X_l = \delta P_l X_l^0 + R_{l+1}^X C_{l+1}^X + R_l^X X_{l-1}$$

So we obtain by recurrence:

$$X_l = C_l^X + D_l^X X_{l-1} \quad (2 \leq l \leq n)$$

with, for  $(2 \leq l < n)$

$$C_l^X = \frac{X_l^0 \delta P_l + R_{l+1}^X C_{l+1}^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

$$D_l^X = \frac{R_l^X}{\delta P_l + R_l^X + R_{l+1}^X (1 - D_{l+1}^X)}$$

## Solving the tridiagonal system

**At the bottom of the boundary layer**  $X_2 = C_2^X + D_2^X X_1$

$$(\delta P_1 + R_2^X) X_1 = \delta P_1 X_1^0 + R_2^X X_2 - g\delta t F_1^X$$

replacing  $X_2$  in the equation above:

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

with

$$A_1^X = \frac{X_1^0 \delta P_1 + R_2^X C_2^X}{\delta P_1 + R_2^X (1 - D_2^X)}$$

$$B_1^X = \frac{-g}{\delta P_1 + R_2^X (1 - D_2^X)}$$

If  $F_1$  is known, then  $X_1$  and all other  $X_i$  are known, and also the flux  $\Phi_i$

# Coupling with the surface

Equation for the variables of the first layer

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

Sensitivity of  $X_1$  to the flux at surface  $F_1$

Value if the flux  $F_1 = 0$

**Atmospheric model:** Boundary layer:

$$X_1 = f(F_1^X)$$

**Surface model:**

Flux between atmosphere and surface:

$$F_1^X = g(X_1, X_s) \Rightarrow F_1^X = g(f(F_1^X), X_s) \Rightarrow F_1^X = \hat{g}(X_s)$$

Surface variables are prescribed or computed (budget at surface):

$$X_s = h(F_1^X) \Rightarrow X_s = h(\hat{g}(X_s)) \Rightarrow X_s = \dots$$

# Coupling with the surface

Equation for the variables of the first layer

$$X_1 = A_1^X + B_1^X \cdot F_1^X \cdot \delta t$$

Sensitivity of  $X_1$  to the flux at surface  $F_1$   
Value if the flux  $F_1=0$

$X$ : temperature  $T$ , humidity  $q$ , velocity  $V_x$  and  $V_y$

$F^x$ : Flux of heat, flux of water mass or flux of momentum

Typical expression of the flux with the surface:  $F_1 = K_1 (X_1 - X_s)$

**Each surface model has to compute  $X_s$  and  $F_1$  using  $X_1$ ,  $A_1$  and  $B_1$**

If  $F_1$  is known, then  $X_1$  and all other  $X_l$  are known, and also the flux  $\Phi_l$

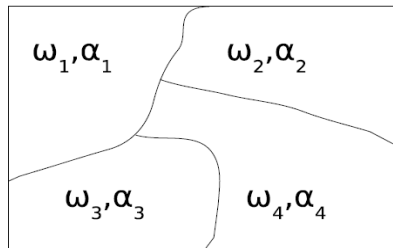
In LMDZ, the turbulent flux are computed separately over each sub-surface type

# Coupling between atmospheric column(s) and sub-surfaces

Each grid cell is divided into several sub-areas or "sub-surfaces" of fractions  $\omega_i$

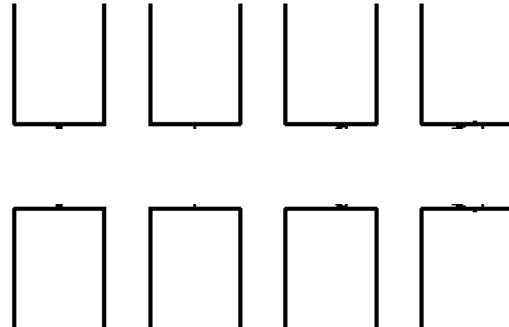
Sub-surfaces

$$\sum_i \omega_i = 1$$



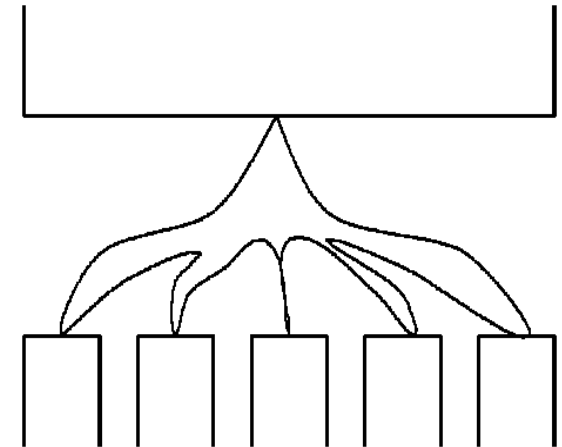
Turbulent flux

One PBL over **each** sub-surface



Radiative flux

One column **covers all** the sub-surface



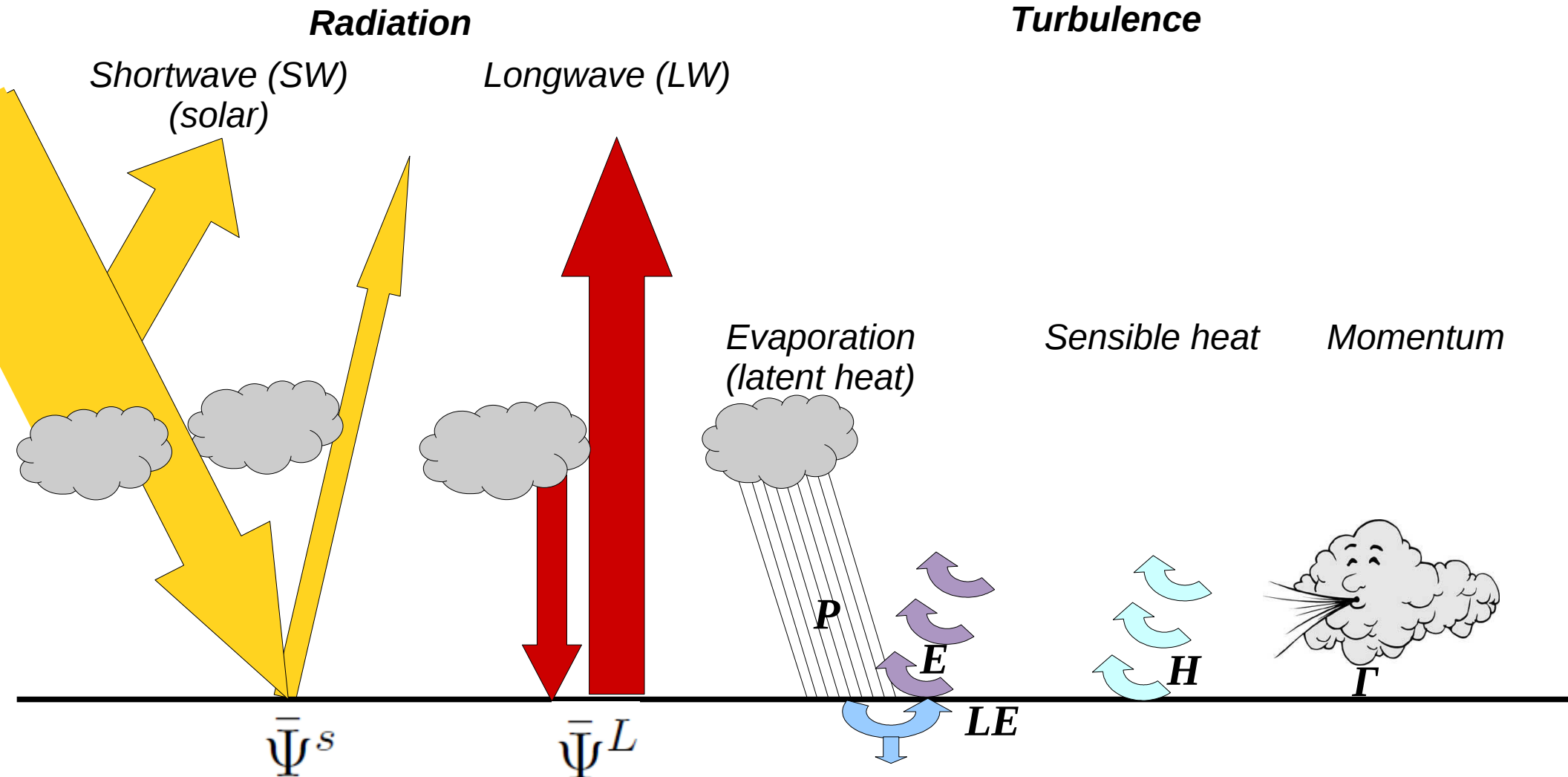
**Each sub surface has to compute  $F_1$  using variables  $X_1, A_1$  and  $B_1$**

The boundary layer tendencies in the atmosphere are mixed between subcolumns (equivalent of averaging the surface flux)

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The surface “receive” precipitation from the atmosphere (no direct feedback).



# Derivation of local sub-surface **net solar radiation** from grid average net solar radiation

The grid average net flux  $\Psi_s$  at surface has been computed for each grid point by the radiative code.

We want (1) to conserve energy and (2) to take into account the value of the local albedo  $\alpha_i$  of the sub-surface.

We compute the downward SW radiation as  $F_{\downarrow}^s = \frac{\Psi_s}{(1 - \alpha)}$

with the mean albedo  $\alpha = \sum_i \omega_i \alpha_i$

**For each sub-surface i**, the absorbed solar radiation reads:  $\psi_i^s = (1 - \alpha_i) F_{\downarrow}^s$

$$\psi_i^s = \frac{(1 - \alpha_i)}{(1 - \alpha)} \bar{\Psi}^s$$

One may verified that this procedure ensure energy conservation, i.e.  $\sum_i \omega_i \psi_i^s = \Psi_s$

# Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

The net longwave (LW) radiation  $\bar{\Psi}^L$  has been computed by the radiative code for each grid cell. How to split it depending on the sub-surfaces local properties and ensuring energy conservation?

If the downward longwave flux  $F_{\downarrow}$  is uniform within each grid, the net LW flux for a sub-surface  $i$  may be written as:

$$\psi_i^L(T_i) = \epsilon_i (F_{\downarrow} - \sigma T_i^4) \quad (1)$$

where  $T_i$  is the surface temperature of sub-surface  $i$  and  $\epsilon_i$  its emissivity. A linearization around the mean temperature  $\bar{T}$  gives:

$$\psi_i^L(T_i) \approx \epsilon_i (F_{\downarrow} - \sigma \bar{T}^4) - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T}) \quad (2)$$

To conserve the energy, the following relationship must be true:

$$\sum_i \omega_i \psi_i^L = \bar{\Psi}^L \quad (3)$$

Using Eq. 2 gives

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i (T_i - \bar{T}) \quad (4)$$

where  $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$  is the mean emissivity.



# Derivation of local sub-surface **net longwave radiation** from grid average net longwave radiation

$$\sum_i \omega_i \psi_i^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) - 4\sigma \bar{T}^3 \sum_i \omega_i \epsilon_i (T_i - \bar{T}) \quad (4)$$

where  $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$  is the mean emissivity. The second term on the right hand side is zero if

$$\bar{T} = \frac{\sum_i \omega_i \epsilon_i T_i}{\bar{\epsilon}} \quad (5)$$

To ensure energy conservation, we need in addition to verify:

$$\bar{\Psi}^L = \bar{\epsilon} (F_{\downarrow} - \sigma \bar{T}^4) \quad (6)$$

Which is consistent with the definition of the net LW flux at the surface. We rewrite now Eq. 2 as:

$$\psi_i^L(T_i) \approx \frac{\epsilon_i}{\bar{\epsilon}} \bar{\Psi}^L - 4\epsilon_i \sigma \bar{T}^3 (T_i - \bar{T}) \quad (7)$$

Due to radiative code limitation, in LMDZ, we always must have  $\epsilon_i = 1$

# Call tree

## In subroutine PHYSIQ

loop over time steps

CALL change\_srf\_frac : Update fraction of the sub-surfaces (pctsrfr)

....

**CALL pbl\_surface** Main subroutine for the interface with surface

Calculate net radiation at sub-surface

*Loop over the sub-surfaces nsrfr*

Compress variables (Consider only one surface type and only the points for which the fraction for this sub-surface is not zero)

CALL clcdrag: coefficients for turbulent diffusion at surface (cdragh and cdragm)

CALL coef\_diff\_turb: coef. turbulent dif. in the atmosphere (ycoefm et ycoefm.)

CALL climb\_hq\_down downhill for henthalpie H and humidity Q

CALL climb\_wind\_down downhill for wind (U and V)

CALL surface models for the various surface types: **surf\_land, surf\_landice, surf\_ocean or surf\_seaice**. **Each surface model computes:**

- evaporation, latent heat flux, sensible heat flux
- surface temperature, albedo

CALL climb\_hq\_up : compute new values of henthalpie H and humidity Q

CALL climb\_wind\_up : compute new values of wind (U and V)

Uncompress variables : (some variables are per unit of sub-surface fraction, some are per unit of grid surface fraction)

Cumulate in global variables after weighting by sub-surface fractions

Surface diagnostics : (T2m, Q2m, wind at 10m...)

*End Loop over the sub-surfaces*

Calculate the mean values over all sub-surfaces for some variables

**End pbl-surface**